CP1 Chapter 7

Linear Transformations

Chapter Overview

1. Use matrices to represent linear transformations

2. Use matrices to represent reflections, rotations (about the origin).

3. Invariant Lines and Points

4. Use matrices to represent enlargements.

5. Carry out successive transformations using matrix products.

6. Use inverse matrices to represent reverse transformations.

3.3	**Use matrices to represent linear transformations in 2-D.** **Successive transformations.** **Single transformations in 3-D.**	**For 2-D, identification and use of the matrix representation of single and combined transformations from: reflection in coordinate axes and lines $y = \pm x$, rotation through any angle about $(0, 0)$, stretches parallel to the x-axis and y-axis, and enlargement about centre $(0, 0)$, with scale factor k, $(k \neq 0)$, where $k \in \mathbb{R}$.** **Knowledge that the transformation represented by AB is the transformation represented by B followed by the transformation represented by A.** **3-D transformations confined to reflection in one of $x = 0, y = 0, z = 0$ or rotation about one of the coordinate axes.** **Knowledge of 3-D vectors is assumed.**
3.4	**Find invariant points and lines for a linear transformation.**	**For a given transformation, students should be able to find the coordinates of invariant points and the equations of invariant lines.**

Linear Transformations

We can use matrices to describe linear transformations. A linear transformation moves all points $(\boldsymbol{x}, \boldsymbol{y})$ in a plane according to some rule.

Transforming a point $\begin{pmatrix} x \\ y \end{pmatrix}$ simply involves multiplying it by some matrix.

From above we can see that multiplying by a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ represents the mapping $T: \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$.

Example

1. Express the linear transformation $T: \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 4x + y \\ x - 2y \end{pmatrix}$ as a matrix.

2. Find matrices to represent these linear transformations.

a) $T: \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2y + x \\ 3x \end{pmatrix}$

b) $V: \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -2y \\ 3x + y \end{pmatrix}$

3. A square has coordinates $(1,1)$, $(3,1)$, $(3,3)$ and $(1,3)$. Find the vertices of the image of S under the transformation given by the matrix $\boldsymbol{M} = \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}$. Sketch S and the image of S on a coordinate grid.