CP1 Chapter 7

Linear Transformations

Chapter Overview

1. Use matrices to represent linear transformations

2. Use matrices to represent reflections, rotations (about the origin).

3. Invariant Lines and Points

4. Use matrices to represent enlargements.

5. Carry out successive transformations using matrix products.

6. Use inverse matrices to represent reverse transformations.



Linear Transformations

We can use matrices to describe linear transformations. A linear transformation moves all points$(x,y)$in a plane according to some rule.

Transforming a point $\left(\begin{matrix}x\\y\end{matrix}\right)$ simply involves multiplying it by some matrix.

From above we can see that multiplying by a matrix $A=\left(\begin{matrix}a&b\\c&d\end{matrix}\right)$ represents the mapping $T:\left(\begin{matrix}x\\y\end{matrix}\right)\rightarrow \left(\begin{matrix}ax+by\\cx+dy\end{matrix}\right)$.

Example

1. Express the linear transformation$    T:\left(\begin{matrix}x\\y\end{matrix}\right)\rightarrow \left(\begin{matrix}4x+y\\x-2y\end{matrix}\right)  $as a matrix.

2. Find matrices to represent these linear transformations.

1. $T:\left(\begin{matrix}x\\y\end{matrix}\right)\rightarrow \left(\begin{matrix}2y+x\\3x\end{matrix}\right)$

b) $V:\left(\begin{matrix}x\\y\end{matrix}\right)\rightarrow \left(\begin{matrix}-2y\\3x+y\end{matrix}\right)$

3. A square has coordinates $\left(1,1\right),\left(3,1\right),(3,3)$ and $(1,3)$. Find the vertices of the image of $S$ under the transformation given by the matrix $M=\left(\begin{matrix}-1&2\\2&1\end{matrix}\right)$. Sketch $S$ and the image of $S$ on a coordinate grid.

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