**7A Introduction to Linear Transformations with Matrices**

Linear Transformations

1. The three transformations **S**, **T** and **U** are defined below. Find the image of the point (2,3) under each of these transformations. State whether each is a *linear* transformation.

$$S:\left[\begin{matrix}x\\y\end{matrix}\right]\rightarrow \left[\begin{matrix}x+4\\y-1\end{matrix}\right]$$

$$T:\left[\begin{matrix}x\\y\end{matrix}\right]\rightarrow \left[\begin{matrix}2x-y\\x+y\end{matrix}\right]$$

$$U:\left[\begin{matrix}x\\y\end{matrix}\right]\rightarrow \left[\begin{matrix}2y\\-x^{2}\end{matrix}\right]$$

Matrices can be used to represent linear transformations:

1. Find matrices to represent these linear transformations:

$$T:\left[\begin{matrix}x\\y\end{matrix}\right]\rightarrow \left[\begin{matrix}2y+x\\3x\end{matrix}\right]$$

$$V:\left[\begin{matrix}x\\y\end{matrix}\right]\rightarrow \left[\begin{matrix}-2y\\3x+y\end{matrix}\right]$$

1. The square S has coordinates (1,1), (3,1), (3,3) and (1,3).

Find the coordinates of the vertices of the image of S after the transformation given by the matrix:

$$M=\left[\begin{matrix}-1&2\\2&1\end{matrix}\right]$$

**7B Reflections & Rotations**

**(0,1)**

**(1,0)**

1. Describe fully the geometrical transformation represented by the matrix:

$$\left[\begin{matrix}3&0\\0&3\end{matrix}\right]$$

$$\left[\begin{matrix}-1&0\\0&-1\end{matrix}\right]$$

$$\left[\begin{matrix}0&-1\\-1&0\end{matrix}\right]$$

1. Find a matrix to represent the transformation:
2. ‘Reflection in the y-axis’
3. ‘Enlargement, centre (0,0), scale factor 2’
4. ‘Rotation of 45° anticlockwise about (0,0)’

As a general rule, the matrix representing a rotation of angle $θ$ anticlockwise about the origin is:

Final notes:

 Invariant point

Invariant Lines

**7E 3D Transformations**

$$z$$

$$x$$

$$y$$

$$z$$

$$x$$

$$y$$

$$z$$

$$y$$

$$x$$

1. A transformation U, in three dimensions, represents a reflection in the plane $z=0$.
2. Write down the $3×3$ matrix that represents this transformation.
3. Find the image of the point $\left(-1,2,3\right)$ under this transformation

Reflection in the $yz$ plane ($x=0)$

Reflection in the $xz$ plane ($y=0)$

Reflection in the $xy$ plane ($z=0)$

1. A transformation U, in three dimensions, represents a 90˚ anticlockwise rotation around the x-axis
2. Write down the $3×3$ matrix that represents this transformation.
3. Find the image of the point $\left(-1,2,3\right)$ under this transformation

Rotation anticlockwise $θ$ around the x-axis

Rotation anticlockwise $θ$ around the y-axis

Rotation anticlockwise $θ$ around the z-axis

1. The matrix $M=\left[\begin{matrix}\frac{\sqrt{3}}{2}&0&\frac{1}{2}\\0&1&0\\-\frac{1}{2}&0&\frac{\sqrt{3}}{2}\end{matrix}\right]$.
2. Describe the transformation represented by $M$.
3. Find the image of the point with coordinates $(-1,-2,1) $under the transformation represented by $M$.

**7C Enlargements**

1. The matrix $M=\left[\begin{matrix}3&0\\0&2\end{matrix}\right]$.
2. Find the image $T^{'}$ of the triangle $T$ with vertices (1,1), (1,2) and (2,2) under the transformation represented by $M$.
3. Sketch $T$ and $T^{'}$ on the same set of coordinate axes.

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-5

1. Describe geometrically the transformation represented by $M$.

**Key note:**

The determinant and scale factors:

1. The Matrix $M=\left[\begin{matrix}2&0\\0&4\end{matrix}\right]$.
2. Describe fully the transformation represented by matrix $M$
3. A triangle $T$ has vertices at (1,0), (4,0) and (4,2). Find the area of the triangle
4. Triangle $T$ is transformed by using matrix $M$. Find the area of the image of $T$.

**7D Multiple Transformations**

1. The points A(1,0), B(0,1) and C(2,0) are the vertices of a triangle T. The triangle T is rotated 90° anticlockwise around (0,0) and then the image T’ is reflected in the line y = x to obtain the triangle T’’.
2. On separate diagrams, draw T, T’ and T’’
3. i) Find the matrix **P** such that **P**(T) = T’

ii) Find the matrix **Q** such that **Q**(T’) = T’’

1. By finding a matrix product, find the single matrix that will perform a 90° anticlockwise rotation followed by a reflection in y = x
2. The following matrices represent three different transformations:

$$R=\left[\begin{matrix}3&7\\-1&-2\end{matrix}\right]$$

$$Q=\left[\begin{matrix}1&2\\0&1\end{matrix}\right]$$

$$P=\left[\begin{matrix}1&1\\2&3\end{matrix}\right]$$

Find the matrix representing the transformation represented by **R**, followed by **Q**, followed by **P** and give a geometrical interpretation of this transformation.

1. $M=\left[\begin{matrix}-2\sqrt{2}&-2\sqrt{2}\\2\sqrt{2}&-2\sqrt{2}\end{matrix}\right]$

The matrix $M$ represents an enlargement with scale factor $k$ followed by an anticlockwise rotation through angle $θ$ about the origin.

1. Find the value of $k$
2. Find the value of $θ$

**7F Inverse Matrices & Transformations**

1. The triangle T has vertices at A, B and C. The matrix:

$$M=\left[\begin{matrix}4&-1\\3&1\end{matrix}\right]$$

transforms T to the triangle T’ with vertices at (4,3), (4,10) and (-4,-3).

Find the coordinates of the points A, B and C

1. The matrix $A=\left[\begin{matrix}2&4\\-2&-5\end{matrix}\right]$ represents a transformation $T$. Given that $T$ maps point $P$ with coordinates (x,y) onto the point $P$’ with coordinates (6,10):
2. Find the coordinates of $P$

The matrix $B$ represents a transformation $U$. Given that the transformation $T$ followed by the transformation $U$ is equivalent to a reflection in the line $y=x$:

1. Find matrix $B$.