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QUESTION SHEET

ALGEBRAIC PROOF OF ARITHMETIC RESULTS

ODD OR EVEN

SAMPLE QUESTIONS 1

(a) Prove algebraically that the sum of two odd numbers is always even.

(b) Prove algebraically that the product of two odd numbers is always odd.

(c) Prove algebraically that the difference between two odd numbers is always even.

(d) Prove algebraically that the difference between an odd number and an even number is always odd.

(e) Prove algebraically algebraically that the square of an even number is always even.

(f) Prove algebraically that $7(n + 2) - 3(n + 4)$ is always even

(g) Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always an even number.

(h) Prove algebraically that $(n + 1)^2 - (n - 1)^2 + 1$ is odd for all positive integer values of n.

(i) Prove algebraically that $(n + 3)(2n + 1) + (n - 2)(2n + 1)$ is **not** an even number

CONSECUTIVE INTEGERS

SAMPLE QUESTIONS 2

(a) Prove algebraically that the sum of any two consecutive odd numbers is always even.

(b) Prove algebraically that the difference between any two consecutive odd numbers is always two.

(c) Prove algebraically that the product of any two consecutive even numbers is always even.

(d) Prove algebraically that the mean of three consecutive integers is always the middle number

(e) Prove algebraically that the sum of any three consecutive even integers is always even.

(f) Prove algebraically that the difference between the squares of any two consecutive integers is always an odd number.

MULTIPLES

SAMPLE QUESTIONS 3

(a) Prove algebraically that $5(n + 7) + 3(n + 3)$ is always a multiple of 4

(b) Prove algebraically that $8(n + 7) – 5(n + 4)$ is always a multiple of 3

(c) Prove algebraically that $7(n + 8) + 5(n – 4)$ is always a multiple of 12

(d) Prove algebraically that $(m + 2)^2 – m^2 – 12$ is always a multiple of 4

(e) Prove algebraically that $(n + 6)^2 – (n + 2)^2$ is always a multiple of 8, for all positive integer values of n.

(f) Prove algebraically that $(5n – 3)^2 – 3(3 – 10n)$ is always a multiple of 5

(g) Prove algebraically that $(4n + 2)^2 – 12(n + 1)$ is always a multiple of 4

(h) Prove algebraically that $(4n + 1)^2 – (4n – 1)^2$ is always a multiple of 8, for all positive integer values of n.

(i) Show that when x is a whole number $7(2x + 1) + 6(x+ 3)$ is always a multiple of 5

(j) Prove algebraically that the sum of three consecutive odd numbers is always a multiple of 3

(k) Prove that $(2n + 1)^2 – (2n – 1)^2 – 2$ is not a multiple of 4 for all positive integer values of n.

MIXED QUESTIONS

(a) Prove algebraically that the sum of the squares of any two odd positive integers is always even.

(b) Prove that the sum of the squares of two consecutive odd numbers is always 2 more than a multiple of 8.

(c) Prove algebraically that the sum of any two consecutive odd numbers is always a multiple of 4.

(d) Prove that the sum of the squares of any three consecutive odd numbers is always 11 more than a multiple of 12.

(e) Prove algebraically that $n^2 − 2 − (n − 2)^2$ is always an even number.

(f) Prove that $(3n + 1)^2 – (3n – 1)^2$ is always a multiple of 12, for all positive integer values of n.

(g) Prove that $(8n + 2)^2 – (8n – 3)^2$ is always a multiple of 5

(h) Prove algebraically that the sum of four consecutive integers is not divisible by 4.

(i) Prove that $(n – 1)^2 + n^2 + (n + 1)^2 = 3n^2 + 2$

EXTENSION 1

(a) Prove that an odd number cubed is also odd.

(b) Prove that the sum of two consecutive multiples of 5 is always an odd number.

(c) Given that $2(x - n) = x + 5$ where n is an integer, prove that x must be an odd number.

(d) Given that $4(x + n) = 3x + 10$ where n is an integer, prove that x must be an even number.

(e) Prove that if the difference of two numbers is 4, then the difference of their squares is a multiple of 8

(f) Prove that for any three consecutive numbers, the difference between the squares of the first and last numbers is 4 times the middle number.

(g) If n is a positive integer greater than 1, prove that $n^3 - n$ is a multiple of 6 for all possible values of n.

(h) Prove that the difference between the squares of any two consecutive integers is equal to the sum of the two integers.

(i) Prove that the sum of the squares of two consecutive odd numbers is always 2 more than a multiple of 8

(j) The product of two consecutive positive integers is added to the larger of the two integers. Prove that the result is always a square number.

EXTENSION 2

(a) If a, b and c are three consecutive numbers, prove that $c^2 - a^2 = 4b$

(b) Given that n is an integer, prove algebraically that the sum of $(n + 2)(n + 1)$ and $n + 2$ is always a square number.

(c) Prove that for any two numbers the product of their difference and their sum is equal to the difference of their squares.

(d) If a, b, c and d are four consecutive integers, show that the product of the first and last is two less than the product of the second and third.

(e) Given that n is an integer, prove that $(n - 2)(n + 3) + (6 - n)$ is a square number.

(f) Given that triangle numbers can be represented by $T_n = \dfrac{n(n+1)}{2}$.

Prove that eight times **<u>any</u>** triangle number is one less than a square number.

(g) n is an integer.

Prove algebraically that the sum of $\frac{1}{2}n(n+1)$ and $\frac{1}{2}(n+1)(n+2)$ is always a square number.

(h) x is a positive whole number.
Explain why the expression $2x^2 + 5x + 2$ can never have a value that is a prime number.

(i) $2^{61} - 1$ is a prime number. Explain why $2^{61} + 1$ must be a multiple of 3.