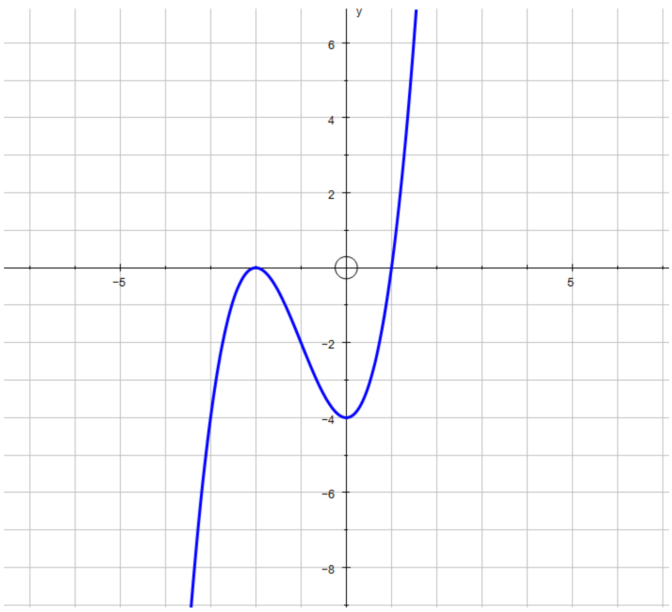
Transformations of Graphs

Task 1: Transforming a graph

If $f(x) = x^3 + 3x^2 - 4$ the graph of $y = f(x)$ should look like this:



It has co-ordinates of (-2,0), (0,-4) and (1,0). For each of the following questions please give the co-ordinates of their new location.

a) If I plot the graph of y = 2f(x), every y value on the original graph is multiplied by 2 for the same x value.

Plot the following graph and copy your result on the axes provided on the following page

$y = 2(x^3 + 3x^2 - 4)$

- notice how the x-interceptions remain the same, there has been a vertical stretch in the y axis of 2.

In general $y = a$ f (x) = Vertical stretch scale factor a (all y coordinates are multiplied by a, points on x axis do not move)

b) If I plot the graph of y = f(2x), every y value on the original graph will be met by an x value of half the size

Plot the following graph and copy your result on the axis below

$y = (2x)^3 + 3(2x)^2 - 4$

- notice how the y-interceptions remain the same, there has been a horizontal stretch in the y axis of 1/2.

In general y = f (a x) = Horizontal stretch scale factor 1/a (all x coordinates are multiplied by $1/a$, points on y axis do not move)

c) If I plot the graph of y = f(x+2), every y value on the original graph will be met by an x value which is 2 less

Plot the following graph and copy your result on the axis below

$y = (x+2)^3 + 3(x+2)^2 - 4$

- notice how the whole graph has translated by two to the left

In general y = f (x+ a) = Translation of a to the left (all x coordinates are moved a to the left)

d) If I plot the graph of y = f(x-2), every y value on the original graph will be met by an x value which is 2 higher

Plot the following graph and copy your result on the axis below

$y = (x-2)^3 + 3(x-2)^2 - 4$

- notice how the whole graph has translated by two to the right

In general y = f (x- a) = Translation of a to the right (all x coordinates are moved a to the right)

e) If I plot the graph of y = f(x) + 2, every y value on the original graph will be 2 more for every x value

Plot the following graph and copy your result on the axis below

$y = (x^3 + 3x^2 - 4) + 2$

- notice how the whole graph has translated by two upwards

In general y = f (x) + a = Translation of a upwards (all y coordinates are moved a upwards)

f) If I plot the graph of y = f(x) - 2, every y value on the original graph will be 2 less for every x value

Plot the following graph and copy your result on the axis below

$y = (x^3 + 3x^2 - 4) - 2$

- notice how the whole graph has translated by two downwards

In general y = f (x) - a = Translation of a down (all y coordinates are moved a down)

<u>Adaptations of rules:</u>

g) If I plot the graph of y = -f(x), every y value on the original graph will now have its sign changed i.e. if it was positive it will now become negative and vice versa

Plot the following graph and copy your result on the axes provided on the following page

$y = -(x^3 + 3x^2 - 4)$

- notice how the x-interceptions remain the same, there has been a reflection in the x axis

In general y = -a f (x) = Vertical stretch scale factor a AND reflect in x axis

h) If I plot the graph of y = f(-x), every y value on the original graph will be met by an x value of which has the opposite sign

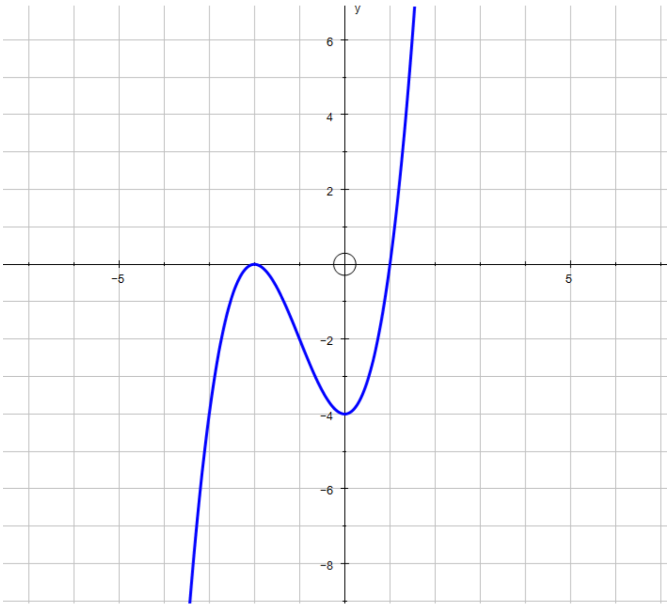
Plot the following graph and copy your result on the axis below

$y = (-x)^3 + 3(-x)^2 - 4$

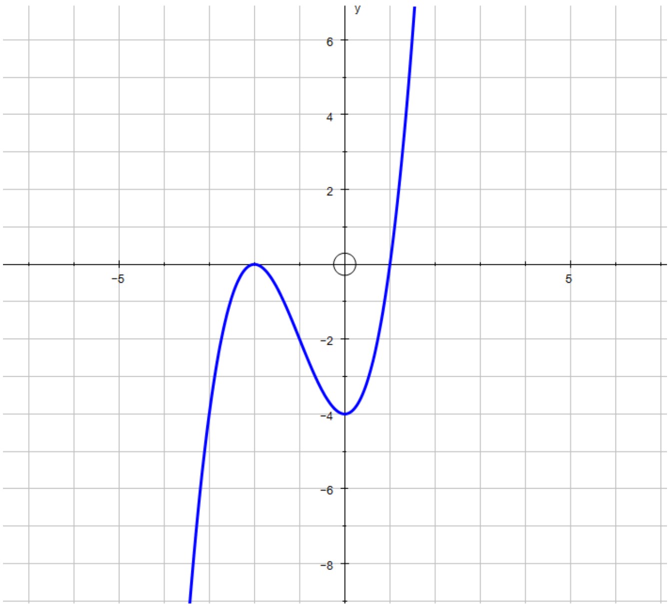
- notice how the y-interceptions remain the same, there has been a reflection in the y axis.

In general y = f (-a x) = Horizontal stretch scale factor 1/a AND reflect in y axis

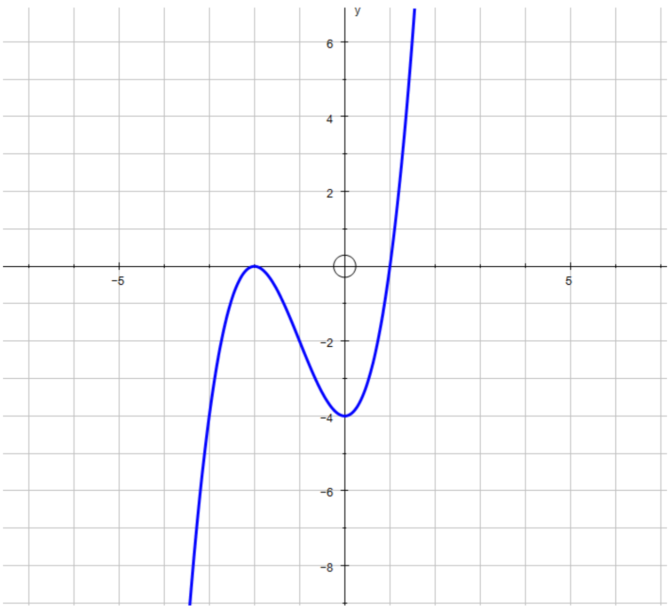
a) y= 2f(x)



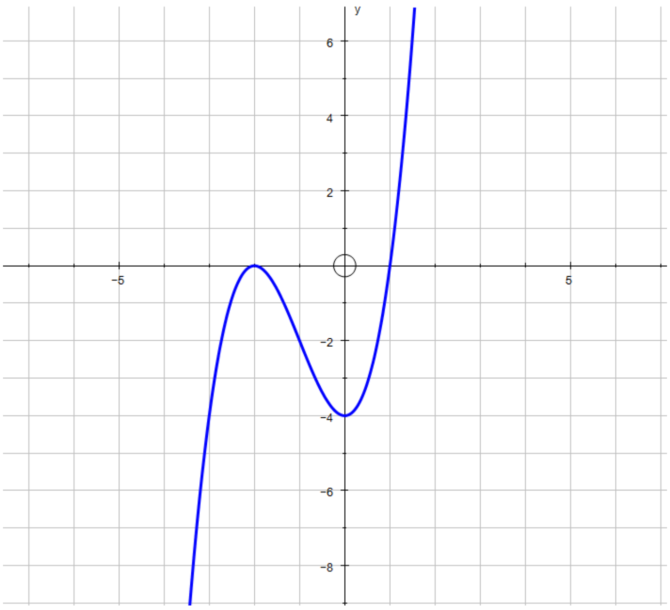
b) y = f(2x)



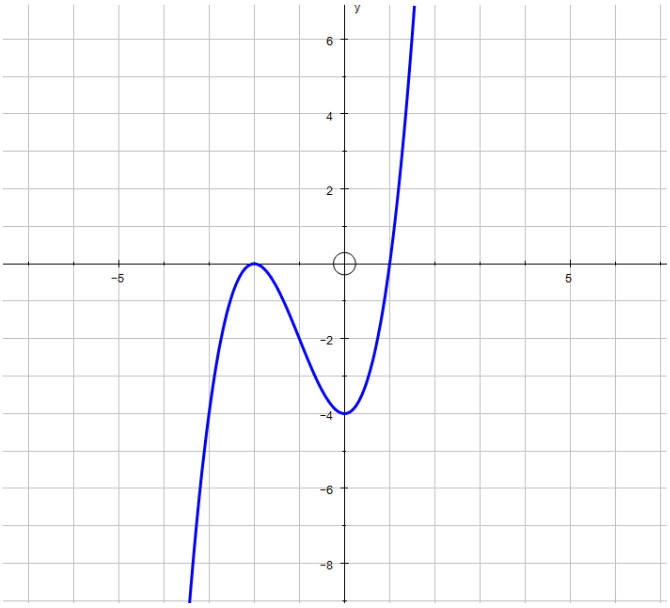
c) y = f(x+2)



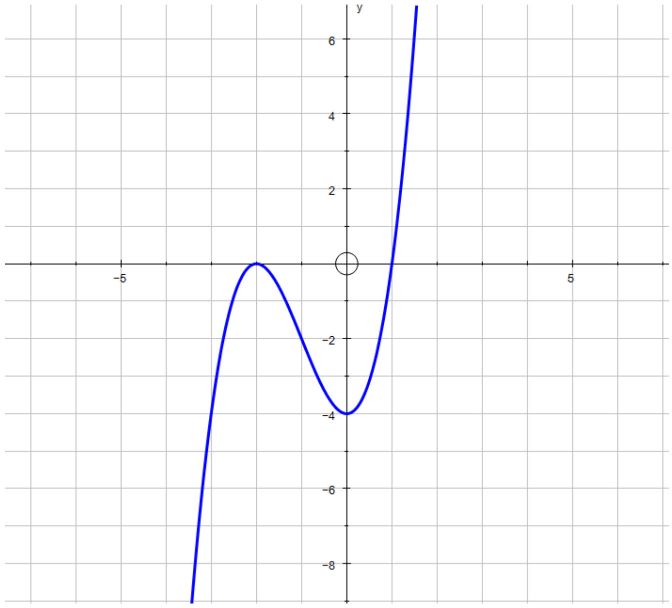
d) y= f(x-2)



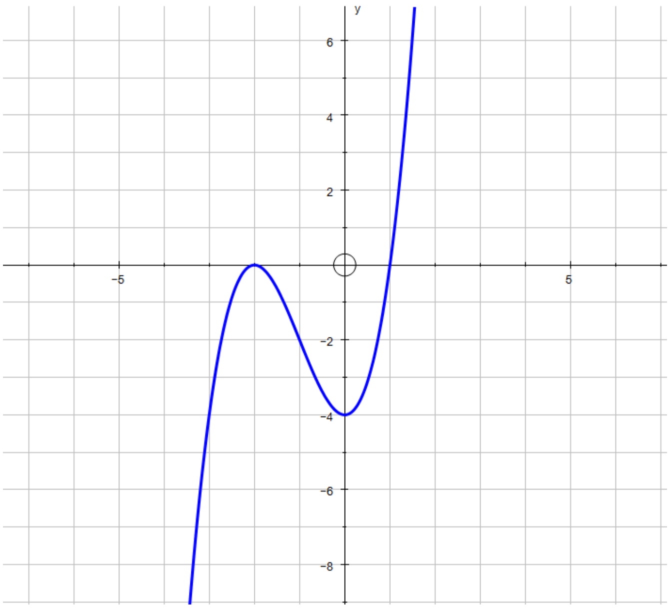
e) y= f(x) +2



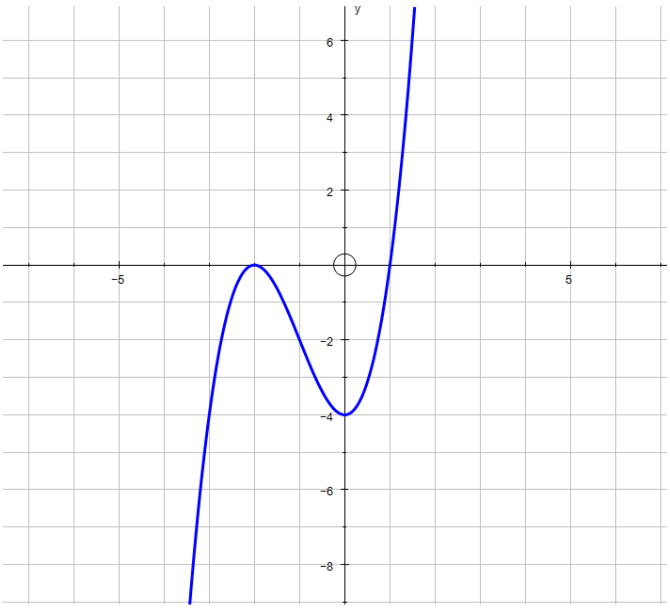
f) y = f(x) - 2



f) y = -f(x)



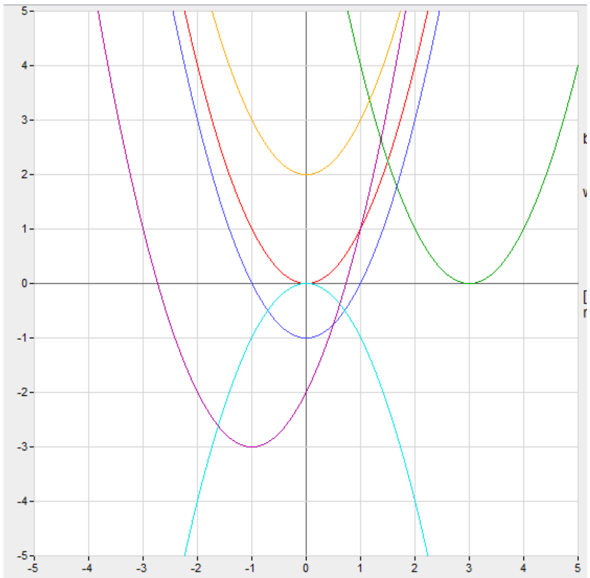
g) y= f(-x)



The 6 main transformations of curves/lines are

1. y=f(x)+a - Translation of $\binom{0}{a}$ (all y coordinates increase by a)
2. y=f(x-a) - Translation of $\binom{a}{0}$ (all x coordinates increase by a)
3. y=af(x) - Vertical stretch scale factor a (all y coordinates are multiplied by a, points on x axis do not move)
4. y=f(ax) - Horizontal stretch scale factor $\frac{1}{a}$ (all x coordinates are divided by a)
5. y=-f(x) - Reflection in x axis (all y coordinates change sign)
6. y=f(-x) - Reflection in y axis (all x coordinates change sign)

You may also have combinations of these.



Task 2: Mainly translations

These graphs have been drawn using a graph plotter. Use Autograph to recreate what you see and then write the equations next to the curves. Some of the equations are given below, the rest you will need to work out for yourselves.

$$y = x^2$$

$$y = (x-3)^2$$

$$y = -x^2$$

$$y = x^2 - 1$$

Task 3 – Mainly stretching

The graphs show y=sin x and one other function. Use a graphing app to recreate what you see and then write the function of the other line on each image.