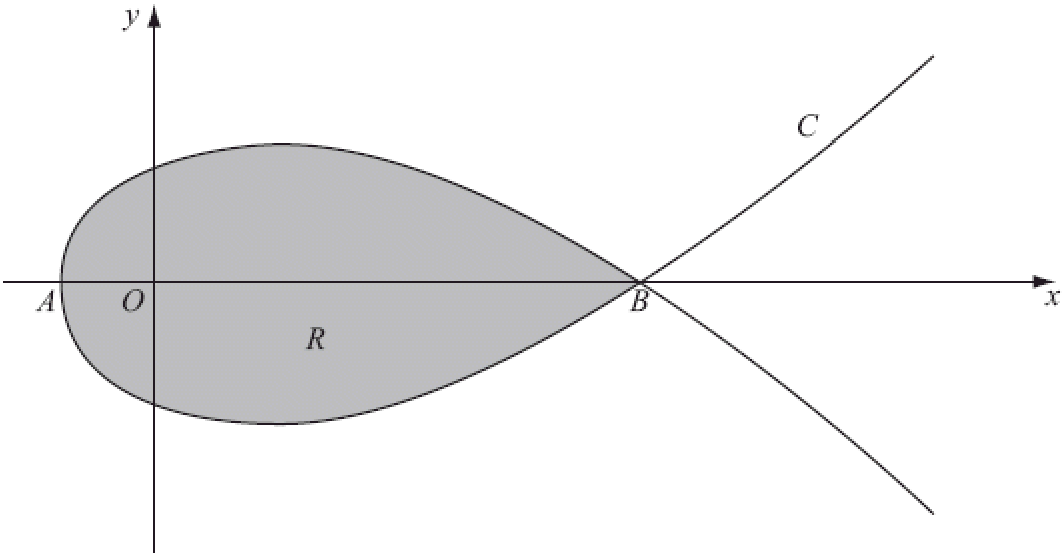
2.



The diagram above shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x-axis at the points A and B.

(a) Find the x-coordinate at the point A and the x-coordinate at the point B.

(3)

The region R, as shown shaded in the diagram above, is enclosed by the loop of the curve.

(b) Use integration to find the area of R.

(6)

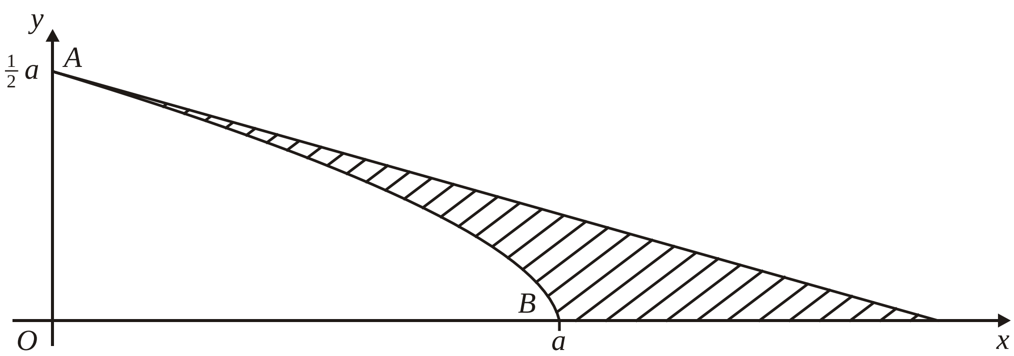
Total 9 marks)

This region is rotated through 2π radians about the *x*-axis.

(b) Find the volume of the solid generated.

(6)
(Total 9 marks)

6.



The curve shown in the figure above has parametric equations

$$x = a\cos 3t,\ y = a\sin t, \quad 0 \le t \le \frac{\pi}{6}.$$

The curve meets the axes at points *A* and *B* as shown.

The straight line shown is part of the tangent to the curve at the point *A*.

Find, in terms of *a*,

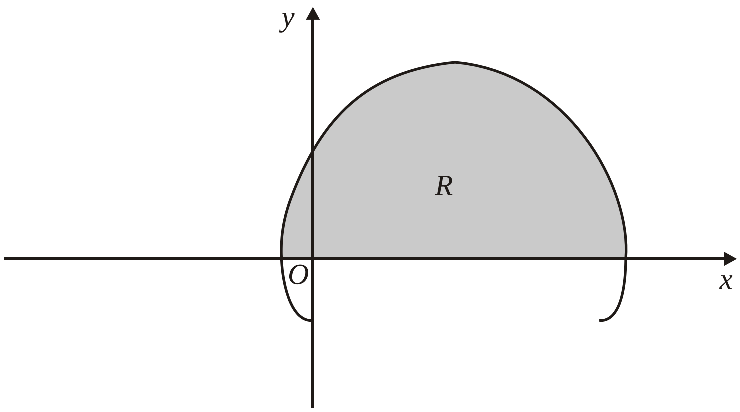
(a) an equation of the tangent at *A*,

(6)

(b) an exact value for the area of the finite region between the curve, the tangent at *A* and the *x*-axis, shown shaded in the figure above.

(9)
(Total 15 marks)

7.



The curve shown in the figure above has parametric equations

$$x = t - 2\sin t, \qquad y = 1 - 2\cos t, \qquad 0 \le t \le 2\pi$$

(a) Show that the curve crosses the x-axis where $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$.

(2)

The finite region R is enclosed by the curve and the x-axis, as shown shaded in the figure above.

(b) Show that the area of R is given by the integral

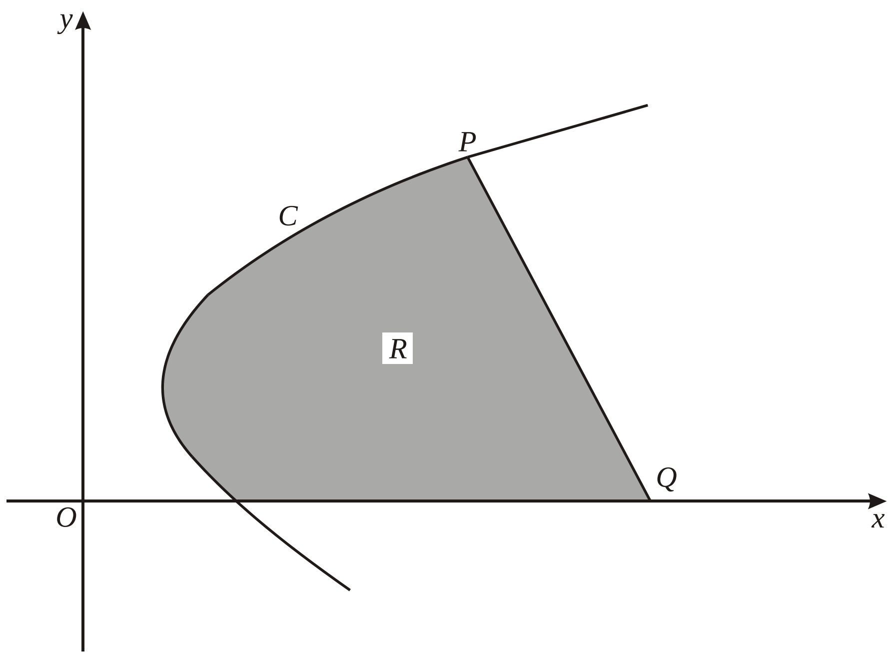
$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 \, dt.$$

(3)

(c) Use this integral to find the exact value of the shaded area.

(7)
(Total 12 marks)

9.



The diagram shows a sketch of part of the curve *C* with parametric equations

$$x = t^2 + 1, \qquad y = 3(1 + t).$$

The normal to *C* at the point *P*(5, 9) cuts the *x*-axis at the point *Q*, as shown in the diagram.

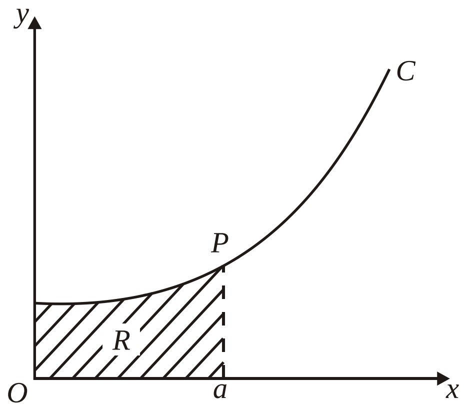
(a) Find the *x*-coordinate of *Q*.

(6)

(b) Find the area of the finite region *R* bounded by *C*, the line *PQ* and the *x*-axis.

(9)
(Total 15 marks)

10.



The diagram above shows a sketch of the curve C with parametric equations

$$x = 3t \sin t,\ y = 2 \sec t, \qquad 0 \le t < \frac{\pi}{2}.$$

The point $P(a, 4)$ lies on C.

(a) Find the exact value of a.

(3)

The region R is enclosed by C, the axes and the line $x = a$ as shown in the diagram above.

(b) Show that the area of R is given by

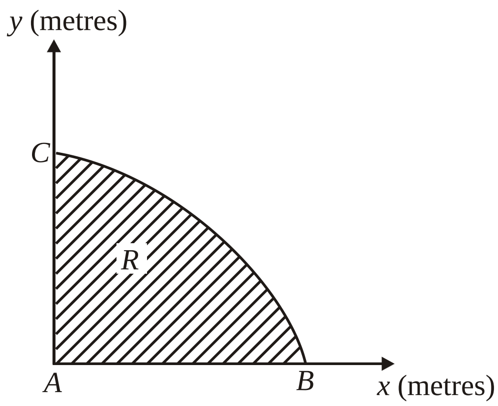
$$6\int_0^{\frac{\pi}{3}} (\tan t + t)\, dt.$$

(4)

(c) Find the exact value of the area of R.

(4)
(Total 11 marks)

11.



The diagram above shows a cross-section R of a dam. The line AC is the vertical face of the dam, AB is the horizontal base and the curve BC is the profile. Taking x and y to be the horizontal and vertical axes, then A, B and C have coordinates (0, 0), $(3\pi^2, 0)$ and (0, 30) respectively. The area of the cross-section is to be calculated.

Initially the profile BC is approximated by a straight line.

(a) Find an estimate for the area of the cross-section R using this approximation.

(1)

The profile BC is actually described by the parametric equations.

$$x = 16t^2 - \pi^2, \quad y = 30 \sin 2t, \quad \frac{\pi}{4} \le t \le \frac{\pi}{2}.$$

(b) Find the exact area of the cross-section R.

(7)

(c) Calculate the percentage error in the estimate of the area of the cross-section R that you found in part (a).

(2)
(Total 10 marks)

1.	(a)	1.386, 2.291	awrt 1.386, 2.291	B1 B1	2
	(b)	$A \approx \frac{1}{2} \times 0.5(...)$		B1	
		= ... (0+2(0.608+1.386+2.291+3.296 +4.385)+5.545)		M1	
		= 0.25(0+2(0.608+1.386+2.291+3.296 +4.385)+5.545)	ft their (a)	A1ft	
		$=0.25 \times 29.477 ... \approx 7.37$	cao	A1	4
	(c)	(i) $\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{1}{x} dx$		M1 A1	
		$= \frac{x^2}{2} \ln x - \int \frac{x}{2} dx$			
		$= \frac{x^2}{2} \ln x - \frac{x^2}{4} (+C)$		M1 A1	
		(ii) $\left[\frac{x^2}{2} \ln x - \frac{x^2}{4}\right]_1^4 = (8 \ln 4 - 4) - \left(-\frac{1}{4}\right)$		M1	
		$=8\ln 4 - \frac{15}{4}$			
		$=8(2\ln 2) - \frac{15}{4}$	ln 4 = 2ln 2 seen or implied	M1	
		$= \frac{1}{4}(64 \ln 2 - 15)$	$a = 64, b = -15$	A1	7

[13]

2.	(a)	$y = 0 \Rightarrow t(9 - t^2) = t(3 - t)(3 + t) = 0$			
		$t = 0, 3, -3$	Any one correct value	B1	
		At $t = 0$, $x = 5(0)^2 - 4 = -4$	Method for finding one value of x	M1	
		At $t = 3$, $x = 5(3)^2 - 4 = 41$			
		(At $t = -3$, $x = 5(-3)^2 - 4 = 41$)			
		At A, $x = -4$; at B, $x = 41$	Both	A1	3

(b) $\frac{dx}{dt} = 10t$ Seen or implied B1

$\int y\,dx = \int y \frac{dx}{dt}\,dt = \int t(9 - t^2)10t\,dt$ M1 A1

$= \int (90t^2 - 10t^2)\,dt$

$\left[\frac{90t^3}{3} - \frac{10t^5}{5}\right]_0^3 = 30 \times 3^3 - 2 \times 3^5 (= 324)$ M1

$A = 2\int y\,dx = 648$ (units^2) A1 6

[9]

3. (a) 1.14805 awrt 1.14805 B1 1

(b) $A \approx \frac{1}{2} \times \frac{3\pi}{8}(...)$ B1

= ... (3 + 2(2.77164 + 2.12132 + 1.14805) + 0) 0 can be implied M1

= $\frac{3\pi}{16}$ (3 + 2(2.77164 + 2.12132 + 1.14805)) ft their (a) A1ft

$\frac{3\pi}{16}$ × 15.08202 ... = 8.884 cao A1 4

(c) $\int 3\cos\left(\frac{x}{3}\right)dx = \frac{3\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$ M1 A1

$= 9\sin\left(\frac{x}{3}\right)$

$A = \left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}} = 9 - 0 = 9$ cao A1 3

[8]

4. (a) Area(R) = $\int_0^2 \frac{3}{\sqrt{(1+4x)}}\,dx = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,dx$

Integrating $3(1 + 4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$. M1

$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$ Correct integration. Ignore limits. A1

$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$

$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$ Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round. M1

$= \frac{9}{2} - \frac{3}{2} = \underline{3}(\text{units})^2$ $\underline{3}$ $\underline{\text{A1}}$ 4

(Answer of 3 with no working scores M0A0M0A0.)

(b) Volume = $\underline{\pi\int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2 \mathrm{d}x}$ Use of $V = \underline{\pi\int y^2 \,\mathrm{d}x}$ .. Can be implied. Ignore limits and dx. B1

$= (\pi)\int_0^2 \frac{9}{1+4x}\,\mathrm{d}x$

$= (\pi)\left[\frac{9}{4}\ln|1+4x|\right]_0^2$ $\pm k\ln|1+4x|$ M1

$\frac{9}{4}\ln|1+4x|$ A1

$= (\pi)\left[(\frac{9}{4}\ln 9) - (\frac{9}{4}\ln 1)\right]$ Substitutes limits of 2 and 0 and subtracts the correct way round. dM1

Note that ln1 can be implied as equal to 0.

So Volume = $\underline{\frac{9}{4}\pi\ln 9}$ $\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$ A1 oe isw 5

Note the answer must be a one term exact value. Note that $\frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.

Note, also you can ignore subsequent working here.

[9]

5. (a) Area Shaded = $\int_0^{2\pi} 3\sin\left(\frac{x}{2}\right)\mathrm{d}x$

$= \left[\frac{-3\cos\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{-2\pi}$

Integrating 3sin($\frac{x}{2}$) to give k cos ($\frac{x}{2}$) with $k \neq 1$. M1

Ignore limits.

$= \left[-6\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$

$-6\cos\left(\frac{x}{2}\right)$ or $\frac{-3}{\frac{1}{2}}\cos\left(\frac{x}{2}\right)$ A1 oe

$= \left[-6(-1)\right]-\left[-6(1)\right]=6+6=\underline{12}$ A1 cao 3

(Answer of 12 with no working scores M0A0A0.)

(b) Volume = $\underline{\pi\int_0^{2\pi}\left(3\sin\left(\frac{x}{2}\right)\right)^2 \quad dx}=9\pi\int_0^{2\pi}\sin^2\left(\frac{x}{2}\right)dx$

Use of $V=\pi\int y^2\,dx$.

Can be implied. Ignore limits. M1

[NB: $\cos 2x = \pm 1 \pm 2\sin^2 x$ gives $\sin^2 x \;\; x=\frac{1-\cos 2x}{2}$]

[NB: $\cos x = \pm 1 \pm 2\sin^2\left(\frac{x}{2}\right)$ gives $\sin^2\left(\frac{x}{2}\right)=\frac{1-\cos x}{2}$] M1

Consideration of the Half Angle Formula for $\sin^2\left(\frac{x}{2}\right)$ *or the Double Angle Formula for* $\sin^2 x$

$\therefore$ Volume = $9(\pi)\int_0^{2\pi}\left(\frac{1-\cos x}{2}\right)dx$ A1

Correct expression for Volume
Ignore limits and π.

$=\frac{9(\pi)}{2}\int_0^{2\pi}(\underline{1-\cos x})\,dx$

$=\frac{9(\pi)}{2}\left[x-\sin x\right]_0^{2\pi}$

Integrating to give $\pm ax \pm b\sin x$; depM1;

Correct integration

$k-k\cos x \to kx-k\sin x$ A1

$=\frac{9\pi}{2}\left[(2\pi-0)-(0-0)\right]$

$=\frac{9\pi}{2}(2\pi)=\underline{9\pi^2}$ or $\underline{88.8264.}$ A1 cso 3

Use of limits to give either $9\pi^2$ *or awrt 88.8*
Solution must be completely correct. No flukes allowed.

[6]

6. (a) $\frac{dx}{dt} = -3a\sin 3t$, $\frac{dy}{dt} = a\cos t$ therefore $\frac{dy}{dx} = \frac{\cos t}{-3\sin 3t}$ M1 A1

When $x = 0$, $t = \frac{\pi}{6}$ B1

Gradient is $-\frac{\sqrt{3}}{6}$ M1

Line equation is $(y - \frac{1}{2}a) = -\frac{\sqrt{3}}{6}(x - 0)$ M1 A1 6

(b) Area beneath curve is $\int a\sin t\,(-3a\sin 3t)dt$ M1

$= -\frac{3a^2}{2}\int(\cos 2t - \cos 4t)dt$ M1

$\frac{3a^2}{2}[\frac{1}{2}\sin 2t - \frac{1}{4}\sin 4t]$ M1 A1

Uses limits 0 and $\frac{\pi}{6}$ to give $\frac{3\sqrt{3}a^2}{16}$ A1

Area of triangle beneath tangent is $\frac{1}{2}\times\frac{a}{2}\times\sqrt{3}a = \frac{\sqrt{3}a^2}{4}$ M1 A1

Thus required area is $\frac{\sqrt{3}a^2}{4} - \frac{3\sqrt{3}a^2}{16} = \frac{\sqrt{3}a^2}{16}$ A1 9

N.B. The integration of the product of two sines is worth 3 marks
(lines 2 and 3 of to part (b))
If they use parts
$\int \sin t\sin 3t\,dt = -\cos t\sin 3t + \int 3\cos 3t\cos t\,dt$ M1

$= -\cos t\sin 3t + 3\cos 3t\sin t + \int 9\sin 3t\sin t\,dt$

$8I = \cos t\sin 3t - 3\cos 3t\sin t$ M1 A1

[15]

7 (a) Solves y = 0 $\Rightarrow$ $\cos t = \frac{1}{2}$ to obtain $t = \frac{\pi}{3}$ *or* $\frac{5\pi}{3}$
(need both for A1) M1 A1 2
Or substitutes **both** values of t and shows that $y = 0$

(b) $\frac{dx}{dt} = 1 - 2\cos t$ M1 A1

Area = $\int y\,dx = \int_{\pi/3}^{5\pi/3} (1 - 2\cos t)\ (1 - 2\cos t)dt$

$= \int_{\pi/3}^{5\pi/3} (1 - 2\cos t)^2\ dt$ AG B1 3

(c) Area $= \int 1 - 4\cos t + 4\cos^2 t\ dt$ 3 terms M1

$\int 1 - 4\cos t + 2(\cos 2t + 1)dt$ (use of correct double angle formula) M1

$= \int 3 - 4\cos t + 2\cos 2t\,dt$ M1 A1

$= [3t - 4\sin t + \sin 2t]$ M1 A1

Substitutes the two correct limits $t = \frac{5\pi}{3}$ and $\frac{\pi}{3}$ and subtracts. M1

$= 4\pi + 3\sqrt{3}$ A1A1 7

[12]

8. (a) $\int xe^{2x}\,dx = \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx$ M1 A1

Attempting parts in the right direction

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}$ A1

$\left[\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right]_0^1 = \frac{1}{4} + \frac{1}{4}e^2$ M1 A1 5

(b) $x = 0.4 \Rightarrow y \approx 0.89022$
$x = 0.8 \Rightarrow y \approx 3.96243$ B1 1

Both are required to 5.d.p.

(c) $I \approx \frac{1}{2} \times 0.2 \times [\ldots]$ B1

$\approx \ldots \times [0 + 7.38906 + 2(0.29836 + .89022 + 1.99207 + 3.96243)]$ M1 A1ft

ft their answers to (b)

$\approx 0.1 \times 21.67522$

≈ 2.168 cao A1 4

Note: $\frac{1}{4} + \frac{1}{4}e^2 \approx 2.097\ldots$

[10]

9. (a) $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{3}{2t}$ M1

Gradient of normal is $-\frac{2t}{3}$ M1

At P $t = 2$ B1

$\therefore$ Gradient of normal @ P is $-\frac{4}{3}$ A1

Equation of normal @ P is $y - 9 = -\frac{4}{3}(x - 5)$ M1

Q is where $y = 0 \therefore x = \frac{27}{4} + 5 = \underline{\frac{47}{4}}$ (o.e.) A1 6

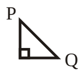
(b) Curved area $= \int y\,dx = \int y \frac{dx}{dt}\,dt$ M1

$= \int 3(1 + b).2t\,dt$ A1

$= [3t^2 + 2t^3]$ M1A1

Curve cuts x-axis when $t = -1$ B1

Curved area $= [3t^2 + 2t^3]_{-1}^{2} = (12 + 16) - (3 - 2)$ (= 27) M1

Area of 

triangle $= \frac{1}{2}((a) - 5) \times 9$ (= 30.375) M1

Total area of R = curved area + Δ M1

= 57.375 or AWRT <u>57.4</u> A1 9

[15]

10.	(a)	$4 = 2\sec t \Rightarrow \cos t = \frac{1}{2}, \Rightarrow t = \frac{\pi}{3}$	M1, A1	
		$\therefore\ a = 3\times\frac{\pi}{3}\times\sin\frac{\pi}{3} = \underline{\frac{\pi\sqrt{3}}{2}}$	B1	3
	(b)	$A = \int_0^a y\,dx = \int y\frac{dx}{dt}\,dt$	M1	
		Change of variable		
		$= \int 2\sec t \times [3\sin t + 3t\cos t]\,dt$	M1	
		Attempt $\frac{dx}{dt}$		
		$= \int_0^{\frac{\pi}{3}} (6\tan t, + 6t)\,dt$ (*)	A1, A1cso	4
		Final A1 requires limit <u>stated</u>		
	(c)	$A = [6\ln\sec t + 3t^2]_0^{\frac{\pi}{3}}$	M1, A1	
		Some integration (M1) both correct (A1) ignore lim.		
		$= (6\ln 2 + 3\times\frac{\pi^2}{9}) - (0)$ Use of $\frac{\pi}{3}$	M1	
		$= \underline{6\ln 2 + \frac{\pi^2}{3}}$	A1	4

[11]

11.	(a)	Area of triangle = $\frac{1}{2}\times 30\times 3\pi^2$ (= 444.132)	B1	1
		Accept 440 or 450		
	(b)	**Either** Area shaded = $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 30\sin 2t.32t\,dt$	M1 A1	
		$= [-480t\cos 2t + \int 480\cos 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	M1 A1	
		$= [-480t\cos 2t + 240\sin 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	A1 ft	
		$= 240(\pi - 1)$	M1 A1	7