7A Cancelling Algebraic Fractions

1. Simplify the following fractions

a) $\frac{7x^4-2x^3+6x}{x}$

b) $\frac{(x+7)(2x-1)}{(2x-1)}$

c) $\frac{x+3}{x^2+7x+12}$

d) $\dfrac{x^2+6x+5}{x^2+3x-10}$

e) $\dfrac{2x^2+11x+12}{(x+3)(x+4)}$

7B Polynomial Division

1. Divide $x^3 + 2x^2 - 17x + 6$ by $(x - 3)$

2. Given that $f(x) = 4x^4 - 17x^2 + 4$, write $f(x)$ in the form:

   $f(x) = (2x + 1)(ax^3 + bx^2 + cx + d)$

3. Find the remainder when $2x^3 - 5x^2 - 16x + 10$ is divided by $(x - 4)$

7C The Factor Theorem

1. Show that $(x-2)$ is a factor of x^3+x^2-4x-4 by:

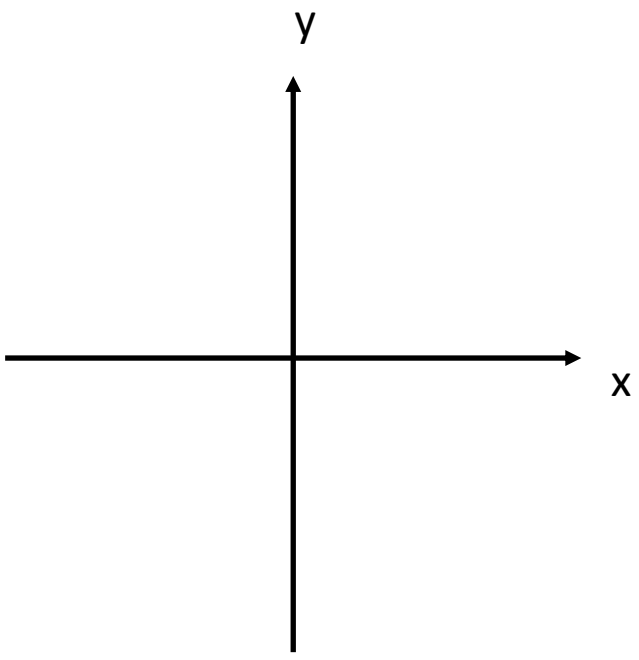
a) Algebraic division

b) The factor theorem

2.

a) Fully factorise $2x^3 + x^2 - 18x - 9$

b) Hence, sketch the graph of $y = 2x^3 + x^2 - 18x - 9$



3. Given that $(x + 1)$ is a factor of $4x^4 – 3x^2 + a$, find the value of a.

7D Algebraic Proof

1. Prove that:

$$(3x + 2)(x - 5)(x + 7) \equiv 3x^3 + 8x^2 - 101x - 70$$

2. Prove that if $(x - p)$ is a factor of $f(x)$ then $f(p) = 0$

3. Prove that $A(1,1)$, $B(3,3)$ and $C(4,2)$ are the vertices of a right-angled triangle.

4. The equation $kx^2 + 3kx + 2 = 0$, where k is a constant, has no real roots. Prove that k satisfies the inequality $0 \leq k < \frac{8}{9}$.

7E Proof by Exhaustion, Counter-Example & Jottings

1. Prove that all square numbers are either a multiple of 4, or 1 more than a multiple of 4

2. Prove that the following statement is not true:

   “The sum of two consecutive prime numbers is always even”

3. Prove that for all positive values of x and y:

$$\frac{x}{y} + \frac{y}{x} \geq 2$$