Differentiation from first principles

Scaffolded solutions

Examples

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

$$for\ f(x) = x^2$$

$$f'(x) = \lim_{h\to 0}\left(\frac{(x+h)^2 - x^2}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left(\frac{x^2 + 2xh + h^2 - x^2}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left(\frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left(\frac{2xh + h^2}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left(\frac{2x\cancel{h} + h^{\cancel{2}}}{\cancel{h}}\right)$$

$$f'(x) = \lim_{h\to 0}(2x + h)$$

$$as\ h \to 0, 2x + h \to 2x$$

$$\therefore f'(x) = 2x$$

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

$$for\ f(x) = x^3$$

$$f'(x) = \lim_{h\to 0}\left(\frac{(x+h)^3 - x^3}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left(\frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left(\frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left(\frac{3x^2h + 3xh^2 + h^3}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left(\frac{3x^2\cancel{h} + 3xh^{\cancel{2}} + h^{\cancel{3}\,2}}{\cancel{h}}\right)$$

$$f'(x) = \lim_{h\to 0}(3x^2 + 3xh + h^2)$$

$$as\ h \to 0, 3xh \to 0, h^2 \to 2x$$

$$\therefore as\ h \to 0, 3x^2 + 3xh + h^2 \to 3x^2$$

$$\therefore f'(x) = 3x^2$$

Question 1

differentiate from first principles x^4

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

$$for\ f(x) = x^4$$

*

$$f'(x) = \lim_{h \to 0}\left(\frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}\right)$$

*

$$f'(x) = \lim_{h \to 0}\left(\frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}\right)$$

*

$$f'(x) = \lim_{h \to 0}(4x^3 + 6x^2h + 4xh^2 + h^3)$$

*

$$\therefore as\ h \to 0, 4x^3 + 6x^2h + 4xh^2 + h^3 \to 4x^3$$

$$\therefore f'(x) = 4x^3$$

Question 2

differentiate from first principles $3x^2$

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

$$for\ f(x) = 3x^2$$

$$f'(x) = \lim_{h \to 0} \left(\frac{3(x+h)^2 - 3x^3}{h} \right)$$

*

$$f'(x) = \lim_{h \to 0} \left(\frac{3(\qquad\qquad\qquad\qquad) - 3x^2}{h} \right)$$

$$f'(x) = \lim_{h \to 0} \left(\frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} \right)$$

*

$$f'(x) = \lim_{h \to 0} \left(\frac{6xh + 3h^2}{h} \right)$$

*

$$f'(x) = \lim_{h \to 0} (6x + 3h)$$

*

*

$$\therefore as\ h \to 0, 6x + 3h \to$$

*

Question 3

differentiate from first principles $5x^3$

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

$$for\ f(x) = 5x^3$$

*

$$f'(x) = \lim_{h \to 0} \left(\frac{5(x^3 + 3x^2h + 3xh^2 + h^3) - 5x^3}{h} \right)$$

*

*

$$f'(x) = \lim_{h \to 0} \left(\frac{15x^2h + 15xh^2 + 5h^2}{h} \right)$$

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*

$$as\ h \to 0, 15xh \to 0\ and\ 5h \to 0$$

*

$$\therefore f'(x) = 15x^2$$

Question 4

differentiate from first principles -2x³

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

$$for\ f(x) = -2x^3$$

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* $$f'(x) = \lim_{h \to 0}\left(\frac{-2(\qquad\qquad) - (\quad)}{h}\right)$$

*

* $$f'(x) = \lim_{h \to 0}\left(\frac{-2x^3 - 6x^2h - 6xh^2 - 2h^3 + 2x^3}{h}\right)$$

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* $as\ h \to 0,$

*

$$\therefore f'(x) = -6x^2$$

Question 5

differentiate from first principles x^4

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$$f'(x) = \lim_{h \to 0}\left(\frac{-7(x+h)^4 - (-7x^4)}{h}\right)$$

*

$$f'(x) = \lim_{h \to 0}\left(\frac{-7x^4 - 28x^3h - 42x^2h^2 - 28xh^3 - 7h^4 + 7x^4}{h}\right)$$

*

*

*

$$f'(x) = \lim_{h \to 0}(-28x^3 - 42x^2h - 28xh^2 - 7h^3)$$

* $as\ h \to 0,$

* $\therefore as\ h \to 0, \quad -28x^3 - 42x^2h - 28xh^2 - 7h^3 \to$

*

Question 6

differentiate from first principles $3x^2 + 7$

*
$f'(x) = \lim_{h \to 0}\left(\frac{3(x+h)^2 + 7 - (3x^2+7)}{h}\right)$
*
$f'(x) = \lim_{h \to 0}\left(\frac{3x^2 + 6xh + 3h^2 + 7 - 3x^2 - 7}{h}\right)$
*
$f'(x) = \lim_{h \to 0}\left(\frac{6xh + 3h^2}{h}\right)$
* $f'(x) = \lim_{h \to 0}\left(\frac{6xh + 3h^2}{h}\right)$
$f'(x) = \lim_{h \to 0}(6x + 3h)$
*
* $\therefore as\ h \to 0,\ 6x + 3h \to$
*

Question 7

differentiate from first principles -3x^3 - 12

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$$f'(x) = \lim_{h \to 0}\left(\frac{-3(x^3 + 3x^2h + 3xh^2 + h^3) - 12 - (-3x^3 - 12)}{h}\right)$$

*

*

*

* $$f'(x) = \lim_{h \to 0}\left(\frac{-9x^2h - 9xh^2 - 3h^3}{h}\right)$$

$$f'(x) = \lim_{h \to 0}(-9x^2 - 9xh - 3h^2)$$

*

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Question 8

differentiate from first principles 5x⁴ - 2x

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

$$for\ f(x) = 5x^4 - 2x$$

$$f'(x) = \lim_{h \to 0}\left(\frac{5(x+h)^4 - 2(x+h) - (5x^4 - 2x)}{h}\right)$$

* $f'(x) = \lim_{h \to 0}\left(\frac{5(\qquad\qquad) - 2(x+h) - (5x^4 - 2x)}{h}\right)$

*

*

$$f'(x) = \lim_{h \to 0}\left(\frac{20x^3h + 30x^2h^2 + 20xh^3 + 5h^4 - 2h}{h}\right)$$

*

*

* $\therefore as\ h \to 0,$

* $\therefore as\ h \to 0,$

* $\therefore f'(x) =$

Questions:

Use Differentiation from first principles to find the derivative of:

a) $6x^2$

b) $8x^3$

c) $-4x^3$

d) $9x$

e) $10x^2-6$

f) $-x^4$

g) $5 - 4x^2$

h) $7x^3 + 6x$

i) $2x^4 - 5x^2$

j) $x^3 + 12x^2 - 7x$