Differentiation from first principles

Scaffolded solutions

Examples



$$∴f^{'}\left(x\right)=3x^{2}$$

$$∴as h\rightarrow 0, 3x^{2}+3xh+h^{2}\rightarrow 3x^{2}$$

$$as h\rightarrow 0, 3xh\rightarrow 0,h^{2}\rightarrow 2x$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}(3x^{2}+3xh+h^{2})$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{3x^{2}h+3xh^{2}+h^{3}}{h}\right)$$

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$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{x^{3}+3x^{2}h+3xh^{2}+h^{3}-x^{3})}{h}\right)$$

$$∴f^{'}\left(x\right)=2x$$

$$as h\rightarrow 0, 2x+h\rightarrow 2x$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}(2x+h)$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{2xh+h^{2}}{h}\right)$$

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$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{x^{3}+3x^{2}h+3xh^{2}+h^{3}-x^{3}}{h}\right)$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{x^{2}+2xh+h^{2}-x^{2}}{h}\right)$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{\left(x+h\right)^{3}-x^{3}}{h}\right)$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{\left(x+h\right)^{2}-x^{2}}{h}\right)$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{f\left(x+h\right)-f(x)}{h}\right)$$

$$for f\left(x\right)= x^{3}$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{f\left(x+h\right)-f(x)}{h}\right)$$

$$for f\left(x\right)= x^{2}$$

Question 1

differentiate from first principles x4

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{f\left(x+h\right)-f(x)}{h}\right)$$

 $for f\left(x\right)= x^{4}$



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{x^{4}+4x^{3}h+6x^{2}h^{2}+4xh^{3}+h^{4}-x^{4}}{h}\right)$$



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{4x^{3}h+6x^{2}h^{2}+4xh^{3}+h^{4}}{h}\right)$$



$$f^{'}\left(x\right)=\lim\_{h\to 0}(4x^{3}+6x^{2}h+4xh^{2}+h^{3})$$



$$∴as h\rightarrow 0, 4x^{3}+6x^{2}h+4xh^{2}+h^{3}\rightarrow 4x^{3}$$

$$∴f^{'}\left(x\right)=4x^{3}$$

Question 2

differentiate from first principles 3x2

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{f\left(x+h\right)-f(x)}{h}\right)$$

 $for f\left(x\right)= 3x^{2}$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{3\left(x+h\right)^{2}-3x^{3}}{h}\right)$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{3( )-3x^{2}}{h}\right)$$



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{3x^{2}+6xh+3h^{2}-3x^{2}}{h}\right)$$



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{6xh+3h^{2}}{h}\right)$$



$$f^{'}\left(x\right)=\lim\_{h\to 0}(6x+3h)$$





$$∴as h\rightarrow 0, 6x+3h\rightarrow $$



Question 3

differentiate from first principles 5x3

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{f\left(x+h\right)-f(x)}{h}\right)$$

 $for f\left(x\right)= 5x^{3}$



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{5(x^{3}+3x^{2}h+3xh^{2}+h^{3} )-5x^{3}}{h}\right)$$



$$∴f^{'}\left(x\right)=15x^{2}$$



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{15x^{2}h+15xh^{2}+5h^{2}}{h}\right)$$





$$as h\rightarrow 0, 15xh\rightarrow 0 and 5h\rightarrow 0$$



Question 4

differentiate from first principles -2x3

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{f\left(x+h\right)-f(x)}{h}\right)$$

 $for f\left(x\right)= -2x^{3}$





$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{-2( )-( )}{h}\right)$$



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{-2x^{3}-6x^{2}h-6xh^{2}-2h^{3}+2x^{3}}{h}\right)$$









$$as h\rightarrow 0, $$





$$∴f^{'}\left(x\right)=-6x^{2}$$

Question 5

differentiate from first principles x4



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{-7\left(x+h\right)^{4}-(-7x^{4})}{h}\right)$$



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{-7x^{4}-28x^{3}h-42x^{2}h^{2}-28xh^{3}-7h^{4}+7x^{4}}{h}\right)$$







$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(-28x^{3}-42x^{2}h-28xh^{2}-7h^{3}\right)$$



$$as h\rightarrow 0, $$



$$∴as h\rightarrow 0, -28x^{3}-42x^{2}h-28xh^{2}-7h^{3}\rightarrow $$



Question 6

differentiate from first principles 3x2 + 7



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{3\left(x+h\right)^{2}+7-(3x^{2}+7)}{h}\right)$$



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{3x^{2}+6xh+3h^{2}+7-3x^{2}-7}{h}\right)$$





$$∴as h\rightarrow 0, 6x+3h\rightarrow $$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{6xh+3h^{2}}{h}\right)$$



$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(6x+3h\right)$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{6xh+3h^{2}}{h}\right)$$



Question 7

differentiate from first principles -3x3 - 12





$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{-3\left(x^{3}+3x^{2}h+3xh^{2}+h^{3}\right)-12-(-3x^{3}-12)}{h}\right)$$











$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{-9x^{2}h-9xh^{2}-3h^{3}}{h}\right)$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(-9x^{2}-9xh-3h^{2}\right)$$





Question 8

differentiate from first principles 5x4 – 2x

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{f\left(x+h\right)-f(x)}{h}\right)$$

 $for f\left(x\right)= 5x^{4}-2x$



$$∴f^{'}\left(x\right)=$$



$$∴as h\rightarrow 0, $$

$$∴as h\rightarrow 0, $$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{20x^{3}h+30x^{2}h^{2}+20xh^{3}+5h^{4}-2h }{h}\right)$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{5( )-2(x+h)-(5x^{4}-2x)}{h}\right)$$

$$f^{'}\left(x\right)=\lim\_{h\to 0}\left(\frac{5(x+h)^{4}-2(x+h)-(5x^{4}-2x)}{h}\right)$$

Questions:

Use Differentiation from first principles to find the derivative of:

1. 6x2
2. 8x3
3. -4x3
4. 9x
5. 10x2-6
6. –x4
7. 5 – 4x2
8. 7x3 + 6x
9. 2x4 – 5x2
10. x3 + 12x2 – 7x