Lower 6 Chapter 8

Binomial Expansion

Chapter Overview

1. Pascal's Triangle
2. Factorial Notation
3. Binomial Expansion
4. Using Expansions for Estimation

4 **Sequences and series**	4.1	**Understand and use the binomial expansion of $(a+bx)^n$ for positive integer n; the notations $n!$ and ${}^nC_r$ link to binomial probabilities.**	**Use of Pascal's triangle.** **Relation between binomial coefficients.** **Also be aware of alternative notations such as $\binom{n}{r}$ and ${}^nC_r$**

Pascal's Triangle:

Starter

a) Expand $(a+b)^0$

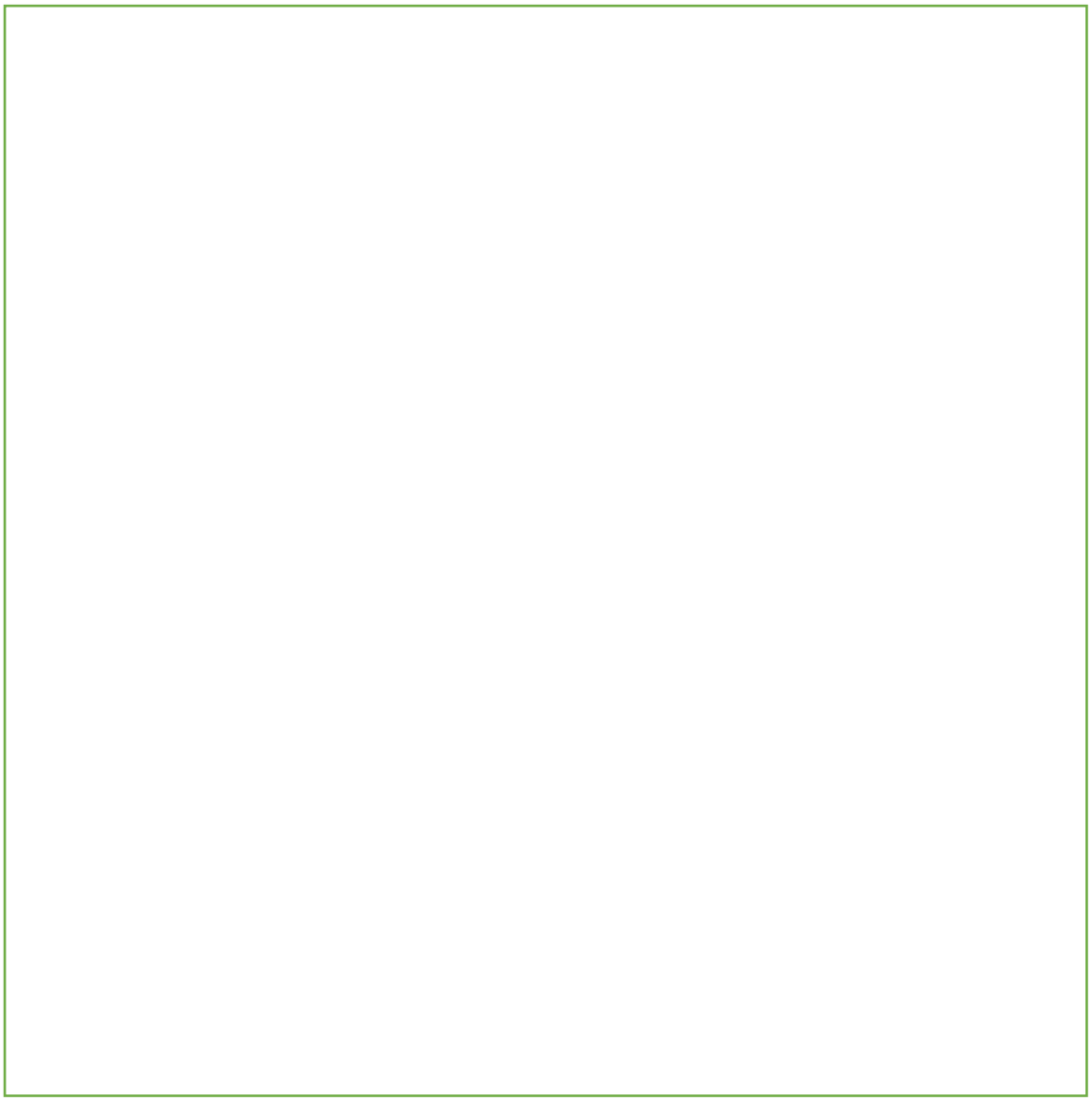
b) Expand $(a+b)^1$

c) Expand $(a+b)^2$

d) Expand $(a+b)^3$

e) Expand $(a+b)^4$

What do you notice about the powers of a and b?



Example

Find the expansion of $(2 + 3x)^4$

Example

Find $(1-2x)^3$ =

Finding a single term example:

The coefficient of x^2 in the expansion of $(2-cx)^5$ is 720.
Find the possible value(s) of the constant c.

Test Your Understanding

(*a*) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a constant. Give each term in its simplest form. **(4)**

Given that the coefficient of x^2 is 6 times the coefficient of x,

(*b*) find the value of k. **(2)**

Extension

[MAT 2009 1J]

The number of pairs of positive integers x, y which solve the equation:

$$x^3 + 6x^2y + 12xy^2 + 8y^3 = 2^{30}$$

is:

A) 0

B) 2^6

C) $2^9 - 1$

D) $2^{10} + 2$

Exercise 8A Page 161

Factorial Notation

Notation:

For example, suppose you had three letters, A, B and C, and wanted to arrange them in a line to form a 'word', e.g. ACB or BAC.

- There are 3 choices for the first letter.
- There are then 2 choices left for the second letter.
- There is then only 1 choice left for the last letter.

There are therefore $3 \times 2 \times 1 = 3! = 6$ possible combinations.

Your calculator can calculate a factorial using the $x!$ button.

For example, if you are a football team captain and need to choose 4 people from amongst 10 in your class, there are $\binom{10}{4} = \frac{10!}{4!6!} = 210$ possible selections.

(Note: the $\binom{10}{4}$ notation is preferable to $10C4$)

Use the nCr button on your calculator (your calculator input should display "10C4")

Examples:

Calculate the value of the following. You may use the factorial button, but not the nCr button.

a) $5!$

b) $\binom{5}{3}$

c) $0!$

d) $\binom{20}{1}$

e) $\binom{20}{0}$

f) $\binom{20}{2}$

g) $\binom{20}{2}$

g) $\binom{20}{18}$

Binomial Expansion

Example

Find the first 4 terms in the expansion of $(3x + 1)^{10}$, in ascending powers of x.

Test Your Understanding

Find the first 3 terms in the expansion of $\left(2-\frac{1}{3}x\right)^7$, in ascending powers of x.

Extension

1. [AEA 2013 Q1a] In the binomial expansion of $\left(1+\frac{12n}{5}x\right)^n$ the coefficients of x^2 and x^3 are equal and non-zero.

Find the possible values of n.

2. [STEP I 2010 Q5a] By considering the expansion of $(1+x)^n$, where n is a positive integer, or otherwise, show that:

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{n}=2^n$$

Finding a Single Term in the Expansion

Expression	Power of x in term wanted.	Term in expansion
$(a+x)^{10}$	3	
$(2x-1)^{75}$	50	
$(3-x)^{12}$	7	
$(3x+4)^{16}$	3	

Example

The coefficient of x^4 in the expansion of $(1+qx)^{10}$ is 3360. Find the possible value(s) of the constant q.

Test Your Understanding

In the expansion of $(1+ax)^{10}$, where a is a non-zero constant the coefficient of x^3 is double the coefficient of x^2. Find the value of a.

Extension

1. *MAT 2014 1G]* Let n be a positive integer. The coefficient of x^3y^5 in the expansion of $(1+xy+y^2)^n$ equals:

A) n

B) 2^n

C) $\binom{n}{3}\binom{n}{5}$

D) $4\binom{n}{4}$

E) $\binom{n}{8}$

2. [STEP I 2013 Q6] By considering the coefficient of x^r in the series for $(1+x)(1+x)^n$, or otherwise, obtain the following relation between binomial coefficients:

$$\binom{n}{r}+\binom{n}{r-1}=\binom{n+1}{r}$$

Exercise 8D Page 166

Using Expansions for Estimating

Example

(*a*) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8,$$

giving each term in its simplest form.

(4)

(*b*) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

Test Your Understanding

(*a*) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(*b*) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

Exercise 8E Page 168