Lower 6 Chapter 8

Binomial Expansion

Chapter Overview

1. Pascal’s Triangle

2. Factorial Notation

3. Binomial Expansion

4. Using Expansions for Estimation



Pascal’s Triangle:

Starter

1. Expand $\left(a+b\right)^{0}$
2. Expand $\left(a+b\right)^{1}$
3. Expand $\left(a+b\right)^{2}$
4. Expand $\left(a+b\right)^{3}$
5. Expand $\left(a+b\right)^{4}$

What do you notice about the powers of a and b?

Example

Find the expansion of $\left(2+3x\right)^{4}$

Example

Find $\left(1-2x\right)^{3}$ =

Finding a single term example:

The coefficient of $x^{2}$ in the expansion of $\left(2-cx\right)^{5}$ is 720. Find the possible value(s) of the constant $c$.

Test Your Understanding



**Extension**

*[MAT 2009 1J]*

The number of pairs of positive integers $x,y$ which solve the equation:

$$x^{3}+6x^{2}y+12xy^{2}+8y^{3}=2^{30}$$

is:

1. 0
2. $2^{6}$
3. $2^{9}-1$

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1. $2^{10}+2$

Factorial Notation

Notation:

For example, suppose you had three letters, A, B and C, and wanted to arrange them in a line to form a ‘word’, e.g. ACB or BAC.

* There are 3 choices for the first letter.
* There are then 2 choices left for the second letter.
* There is then only 1 choice left for the last letter.

There are therefore $3×2×1=3!=6$ possible combinations.

**Your calculator can calculate a factorial using the** $x!$ **button.**

For example, if you are a football team captain and need to choose 4 people from amongst 10 in your class, there are $\left(\begin{matrix}10\\4\end{matrix}\right)=\frac{10!}{4!6! }=210$ possible selections.

*(Note: the* $\left(\begin{matrix}10\\4\end{matrix}\right)$ *notation is preferable to* $10C4$*)*

**Use the** $nCr$ **button on your calculator (your calculator input should display “10C4”)**

Examples:

Calculate the value of the following. You may use the factorial button, but not the nCr button.

1. $5!$ b) $\left(\begin{matrix}5\\3\end{matrix}\right)$
2. $0!$ d) $\left(\begin{matrix}20\\1\end{matrix}\right)$

e) $\left(\begin{matrix}20\\0\end{matrix}\right)$ f) $\left(\begin{matrix}20\\2\end{matrix}\right)$

g) $\left(\begin{matrix}20\\2\end{matrix}\right)$ g) $\left(\begin{matrix}20\\18\end{matrix}\right)$

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Binomial Expansion

Example

Find the first 4 terms in the expansion of $\left(3x+1\right)^{10}$, in ascending powers of $x$.

Test Your Understanding

Find the first 3 terms in the expansion of $\left(2-\frac{1}{3}x\right)^{7}$, in ascending powers of $x$.

Extension

1. [AEA 2013 Q1a] In the binomial expansion of $\left(1+\frac{12n}{5}x\right)^{n}$ the coefficients of $x^{2}$ and $x^{3}$ are equal and non-zero.

Find the possible values of $n$.

2. [STEP I 2010 Q5a] By considering the expansion of $\left(1+x\right)^{n}$, where $n$ is a positive integer, or otherwise, show that:

$$\left(\begin{matrix}n\\0\end{matrix}\right)+\left(\begin{matrix}n\\1\end{matrix}\right)+\left(\begin{matrix}n\\2\end{matrix}\right)+…+\left(\begin{matrix}n\\n\end{matrix}\right)=2^{n}$$

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Finding a Single Term in the Expansion



Example

The coefficient of $x^{4}$ in the expansion of $\left(1+qx\right)^{10}$ is 3360. Find the possible value(s) of the constant $q$.

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In the expansion of $\left(1+ax\right)^{10}$, where $a$ is a non-zero constant the coefficient of $x^{3}$ is double the coefficient of $x^{2}$. Find the value of $a$.

Extension

1. *MAT 2014 1G]* Let $n$ be a positive integer. The coefficient of $x^{3}y^{5}$ in the expansion of $\left(1+xy+y^{2}\right)^{n}$ equals:
2. $n$
3. $2^{n}$
4. $\left(\begin{matrix}n\\3\end{matrix}\right)\left(\begin{matrix}n\\5\end{matrix}\right)$
5. $4\left(\begin{matrix}n\\4\end{matrix}\right)$
6. $\left(\begin{matrix}n\\8\end{matrix}\right)$
7. [STEP I 2013 Q6] By considering the coefficient of $x^{r}$ in the series for $\left(1+x\right)\left(1+x\right)^{n}$, or otherwise, obtain the following relation between binomial coefficients:

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$$\left(\begin{matrix}n\\r\end{matrix}\right)+\left(\begin{matrix}n\\r-1\end{matrix}\right)=\left(\begin{matrix}n+1\\r\end{matrix}\right)$$

Using Expansions for Estimating

Example

Test Your Understanding

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