Lower 6 Chapter 7

Algebraic Methods

Chapter Overview

1. Algebraic Fractions

2. Algebraic Long Division

3. Factor Theorem

4. Proof





Simplifying Algebraic Fractions

Examples

1. 2.

3. 4.

Exercise 7A Page 138

Algebraic Long Division

Examples

1.

2.

Test your understanding

1. Find the remainder when is divided by .

2. Divide by .

Exercise 7B Page 141

The Factor Theorem

Examples

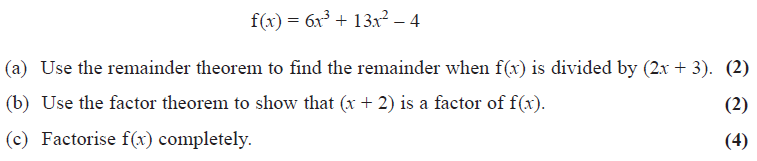
1. Show that is a factor of .

2. Fully factorise .

Using the factor theorem to find unknown coefficients:

1. Given that is a factor of , determine the value of .

Test your understanding



2. Given that is a factor of , determine the value of .

Extension

1. *[MAT 2006 1E]* The cubic has both and has factors. Determine the values of and .

2. [MAT 2009 1I] The polynomial has as a factor

1. for no values of ;
2. for only;
3. for only;
4. for and only.

The **remainder** **theorem** states that if is divided by , the remainder is . This similarly works whenever makes the divisor 0.

(No longer required for A Level)

3. [MAT 2013 1G] Let be an integer and be the polynomial

What is the remainder, in terms of , when is divided by ?

Exercise 7C Page 145

Proof

* A **conjecture** is a mathematical statement that has yet to be proven.
* A **theorem** is a mathematical statement that has been proven.

Proof by Deduction:

This is the simplest type, where you start from known facts and reach the desired conclusion via deductive steps.

Examples:

1. **“Prove that the product of two odd numbers is odd.”**

2. **“Prove that ”**

3. **Prove that if three consecutive integers are the sides of a right-angled triangle, they must be 3, 4 and 5**

Test your Understanding:

**Prove that the sum of the squares of two consecutive odd numbers is 2 more than a multiple of 8.**

**Extension**

*[STEP I 2005 Q1]* 47231 is a five-digit number whose digits sum to

.

1. Prove that there are 15 five-digit numbers whose digits sum to 43. You should explain your reasoning clearly.
2. How many five-digit numbers are there whose digits sum to 39?

Exercise 7D Page 149

Proof by Exhaustion

Example: **Prove that is even for all integers .**

Disproof by counter-example

Example: **Disprove the statement:**

**“ is prime for all integers .”**

**[Proof by contradiction covered in Year 2]**

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