Lower 6 Chapter 7

Algebraic Methods

Chapter Overview

1. Algebraic Fractions

2. Algebraic Long Division

3. Factor Theorem

4. Proof





Simplifying Algebraic Fractions

Examples

1. $\frac{x^{2}-1}{x^{2}+x}$ 2. $\frac{x^{2}+3x+2}{x+1}$

3. $\frac{2x^{2}+11x+12}{x^{2}+9x+20}$ 4. $\frac{4-x^{2}}{x^{2}+2x-8}$

Exercise 7A Page 138

Algebraic Long Division

Examples

1.

2.

Test your understanding

1. Find the remainder when $2x^{3}-5x^{2}-16x+10$ is divided by $x-4$.

2. Divide $8x^{3}-1$ by $2x-1$.

Exercise 7B Page 141

The Factor Theorem

Examples

1. Show that $(x-2)$ is a factor of $x^{3}+x^{2}-4x-4$.

2. Fully factorise $2x^{3}+x^{2}-18x-9$.

Using the factor theorem to find unknown coefficients:

1. Given that $2x+1$ is a factor of $6x^{3}+ax^{2}+1$, determine the value of $a$.

Test your understanding



2. Given that $3x-1$ is a factor of $3x^{3}+11x^{2}+ax+1$, determine the value of $a$.

Extension

1. *[MAT 2006 1E]* The cubic $x^{3}+ax+b$ has both $\left(x-1\right)$ and $\left(x-2\right)$ has factors. Determine the values of $a$ and $b$.

2. [MAT 2009 1I] The polynomial $n^{2}x^{2n+3}-25nx^{n+1}+150x^{7}$ has $x^{2}-1$ as a factor

1. for no values of $n$;
2. for $n=10$ only;
3. for $n=15$ only;
4. for $n=10$ and $n=15$ only.

The **remainder** **theorem** states that if $f(x)$ is divided by $(x-a)$, the remainder is $f(a)$. This similarly works whenever $a$ makes the divisor 0.

(No longer required for A Level)

3. [MAT 2013 1G] Let $n\geq 2$ be an integer and $p\_{n}\left(x\right)$ be the polynomial

$$p\_{n}\left(x\right)=\left(x-1\right)+\left(x-2\right)+…+(x-n)$$

What is the remainder, in terms of $n$, when $p\_{n}\left(x\right)$ is divided by $p\_{n-1}\left(x\right)$?

Exercise 7C Page 145

Proof

* A **conjecture** is a mathematical statement that has yet to be proven.
* A **theorem** is a mathematical statement that has been proven.

Proof by Deduction:

This is the simplest type, where you start from known facts and reach the desired conclusion via deductive steps.

Examples:

1. **“Prove that the product of two odd numbers is odd.”**

2. **“Prove that** $\left(3x+2\right)\left(x-5\right)\left(x+7\right)≡3x^{3}+8x^{2}-101x-70$**”**

3. **Prove that if three consecutive integers are the sides of a right-angled triangle, they must be 3, 4 and 5**

Test your Understanding:

**Prove that the sum of the squares of two consecutive odd numbers is 2 more than a multiple of 8.**

**Extension**

*[STEP I 2005 Q1]* 47231 is a five-digit number whose digits sum to

$4+7+2+3+1=17$.

1. Prove that there are 15 five-digit numbers whose digits sum to 43. You should explain your reasoning clearly.
2. How many five-digit numbers are there whose digits sum to 39?

Exercise 7D Page 149

Proof by Exhaustion

Example: **Prove that** $n^{2}+n$ **is even for all integers** $n$**.**

Disproof by counter-example

Example: **Disprove the statement:**

**“**$n^{2}-n+41$ **is prime for all integers** $n$**.”**

**[Proof by contradiction covered in Year 2]**

Exercise 7E Page 152