

## Proof

- A **conjecture** is a mathematical statement that has yet to be proven.
- A **theorem** is a mathematical statement that has been proven.

### Proof by Deduction:

Examples:

1. **“Prove that the product of two odd numbers is odd.”**

2. **“Prove that  $(3x + 2)(x - 5)(x + 7) \equiv 3x^3 + 8x^2 - 101x - 70$ ”**

**3. Prove that if three consecutive integers are the sides of a right-angled triangle, they must be 3, 4 and 5**

Test your Understanding:

**Prove that the sum of the squares of two consecutive odd numbers is 2 more than a multiple of 8.**

## Extension

[STEP 1 2005 Q1] 47231 is a five-digit number whose digits sum to

$$4 + 7 + 2 + 3 + 1 = 17.$$

- (i) Prove that there are 15 five-digit numbers whose digits sum to 43. You should explain your reasoning clearly.
- (ii) How many five-digit numbers are there whose digits sum to 39?

Proof by Exhaustion

Example: **Prove that  $n^2 + n$  is even for all integers  $n$ .**

Disproof by counter-example

Example: **Disprove the statement:**

**“ $n^2 - n + 41$  is prime for all integers  $n$ .”**

**[Proof by contradiction covered in Year 2]**