Proof

* A **conjecture** is a mathematical statement that has yet to be proven.
* A **theorem** is a mathematical statement that has been proven.

Proof by Deduction:

This is the simplest type, where you start from known facts and reach the desired conclusion via deductive steps.

Examples:

1. **“Prove that the product of two odd numbers is odd.”**

2. **“Prove that** $\left(3x+2\right)\left(x-5\right)\left(x+7\right)≡3x^{3}+8x^{2}-101x-70$**”**

3. **Prove that if three consecutive integers are the sides of a right-angled triangle, they must be 3, 4 and 5**

Test your Understanding:

**Prove that the sum of the squares of two consecutive odd numbers is 2 more than a multiple of 8.**

**Extension**

*[STEP I 2005 Q1]* 47231 is a five-digit number whose digits sum to

$4+7+2+3+1=17$.

1. Prove that there are 15 five-digit numbers whose digits sum to 43. You should explain your reasoning clearly.
2. How many five-digit numbers are there whose digits sum to 39?

Exercise 7D Page 149

Proof by Exhaustion

Example: **Prove that** $n^{2}+n$ **is even for all integers** $n$**.**

Disproof by counter-example

Example: **Disprove the statement:**

**“**$n^{2}-n+41$ **is prime for all integers** $n$**.”**

**[Proof by contradiction covered in Year 2]**

Exercise 7E Page 152