

Lower 6 Chapter 7

Algebraic Methods

Chapter Overview

1. Algebraic Fractions
2. Algebraic Long Division
3. Factor Theorem
4. Proof

2
Algebra and functions
continued

2.6 **Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem.**

Simplify rational expressions, including by factorising and cancelling, and algebraic division (by linear expressions only).

Only division by $(ax + b)$ or $(ax - b)$ will be required. Students should know that if $f(x) = 0$ when $x = a$, then $(x - a)$ is a factor of $f(x)$.

Students may be required to factorise cubic expressions such as $x^3 + 3x^2 - 4$ and $6x^3 + 11x^2 - x - 6$.

Denominators of rational expressions will be linear or quadratic,

e.g. $\frac{1}{ax+b}, \frac{ax+b}{px^2+qx+r}, \frac{x^3+a^3}{x^2-a^2}$

1
Proof

1.1 **Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including:**

Proof by deduction

Proof by exhaustion

Disproof by counter example

Examples of proofs:

Proof by deduction

e.g. using completion of the square, prove that $n^2 - 6n + 10$ is positive for all values of n or, for example, differentiation from first principles for small positive integer powers of x or proving results for arithmetic and geometric series. **This is the most commonly used method of proof throughout this specification**

Proof by exhaustion

This involves trying all the options. Suppose x and y are odd integers less than 7. Prove that their sum is divisible by 2.

Disproof by counter example

e.g. show that the statement " $n^2 - n + 1$ is a prime number for all values of n " is untrue

Simplifying Algebraic Fractions



Examples

$$1. \frac{x^2-1}{x^2+x}$$

$$2. \frac{x^2+3x+2}{x+1}$$

$$3. \frac{2x^2+11x+12}{x^2+9x+20}$$

$$4. \frac{4-x^2}{x^2+2x-8}$$

Algebraic Long Division

Examples

1.

2.

Test your understanding

1. Find the remainder when $2x^3 - 5x^2 - 16x + 10$ is divided by $x - 4$.

2. Divide $8x^3 - 1$ by $2x - 1$.

The Factor Theorem



Examples

1. Show that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$.

2. Fully factorise $2x^3 + x^2 - 18x - 9$.

Using the factor theorem to find unknown coefficients:

1. Given that $2x + 1$ is a factor of $6x^3 + ax^2 + 1$, determine the value of a .

Test your understanding

$$f(x) = 6x^3 + 13x^2 - 4$$

- (a) Use the remainder theorem to find the remainder when $f(x)$ is divided by $(2x + 3)$. (2)
- (b) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$. (2)
- (c) Factorise $f(x)$ completely. (4)

2. Given that $3x - 1$ is a factor of $3x^3 + 11x^2 + ax + 1$, determine the value of a .

Extension

1. [MAT 2006 1E] The cubic $x^3 + ax + b$ has both $(x - 1)$ and $(x - 2)$ as factors. Determine the values of a and b .

2. [MAT 2009 1I] The polynomial $n^2x^{2n+3} - 25nx^{n+1} + 150x^7$ has $x^2 - 1$ as a factor

- A) for no values of n ;
- B) for $n = 10$ only;
- C) for $n = 15$ only;
- D) for $n = 10$ and $n = 15$ only.

The **remainder theorem** states that if $f(x)$ is divided by $(x - a)$, the remainder is $f(a)$. This similarly works whenever a makes the divisor 0.

(No longer required for A Level)

3. [MAT 2013 1G] Let $n \geq 2$ be an integer and $p_n(x)$ be the polynomial

$$p_n(x) = (x - 1) + (x - 2) + \cdots + (x - n)$$

What is the remainder, in terms of n , when $p_n(x)$ is divided by $p_{n-1}(x)$?