Lower 6 Chapter 6

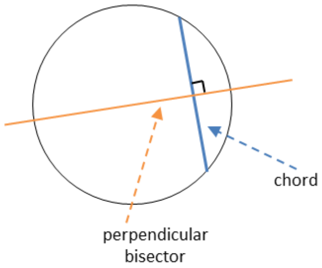
Circles

Chapter Overview

1. Perpendicular bisector recap
2. Equations of circles
3. Intersections of lines and circles
4. Chords, tangents and perpendicular bisectors
5. Circumscribing Triangles

3.2	**Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$**	**Students should be able to find the radius and the coordinates of the centre of the circle given the equation of the circle, and vice versa.** **Students should also be familiar with the equation $x^2 + y^2 + 2fx + 2gy + c = 0$**
	Completing the square to find the centre and radius of a circle; use of the following properties:	
	• **the angle in a semicircle is a right angle** • **the perpendicular from the centre to a chord bisects the chord**	**Students should be able to find the equation of a circumcircle of a triangle with given vertices using these properties.**
	• **the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point.**	**Students should be able to find the equation of a tangent at a specified point, using the perpendicular property of tangent and radius.**

Perpendicular bisectors and mid-points



Example:

Find the equation of the perpendicular bisector of A (2,5) and B (6,7).

Test Your Understanding:

1. Find the perpendicular bisector of the line AB where A and B have the coordinates:

   a) $A(4,7)$, $B(10,17)$

2. A line segment AB is the diameter of a circle with centre $(5, -4)$. If A has coordinates $(1, -2)$, what are the coordinates of B?

Exercise 6A/B
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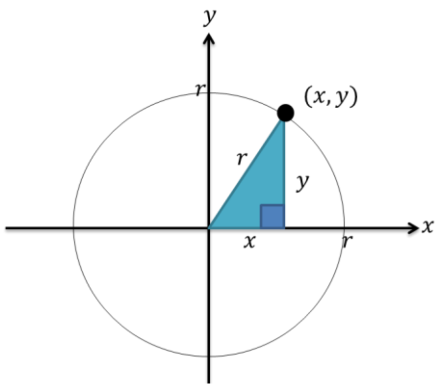
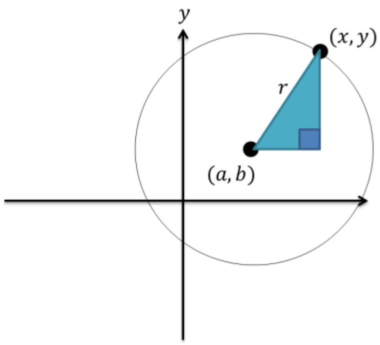
Equation of a circle





Examples:

1.

Centre	Radius	Equation
(0,0)	5	
(1,2)	6	
		$(x+3)^2+(y-5)^2=1$
		$(x+5)^2+(y-2)^2=49$
		$(x+6)^2+y^2=16$

		$(x-1)^2+(y+1)^2=3$
		$(x+2)^2+(y-3)^2=8$

2. A line segment AB is the diameter of a circle, where A and B have coordinates $(5,8)$ and $(-7,4)$ respectively. Determine the equation of the circle.

Test your understanding

The points A and B have coordinates $(5, -1)$ and $(13, 11)$ respectively.

(*a*) Find the coordinates of the mid-point of AB.

(2)

Given that AB is a diameter of the circle C,

(*b*) find an equation for C.

(4)

Completing the Square

Example

Find the centre and radius of the circle with equation $x^2 + y^2 - 6x + 2y - 6 = 0$

Test your understanding

The circle C with centre T and radius r has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

(a) Find the coordinates of the centre of C.

(3)

(b) Show that $r = 5$

(2)

Extension:

1. [MAT 2009 1B] The point on the circle $x^2 + y^2 + 6x + 8y = 75$ which is closest to the origin, is at what distance from the origin?

2. [MAT 2007 1D]

The point on the circle $(x-5)^2 + (y-4)^2 = 4$ which is closest to the circle $(x-1)^2 + (y-1)^2 = 1$ has what coordinates?

3. [MAT 2016 1I] Let a and b be positive real numbers. If $x^2 + y^2 \leq 1$ then the largest that $ax + by$ can equal is what?

Give your expression in terms of a and b.

Exercise 6C Page 119 - 120

The Intersection of Lines and Circles

Example: Show that the line $y = x + 3$ never intersects the circle with equation

$x^2 + y^2 = 1$.

Test your understanding:

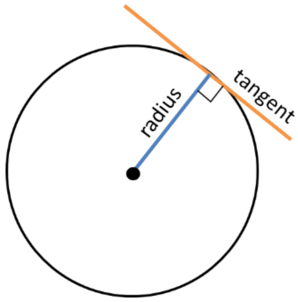
1. Find the points of intersection where the line $y = x + 6$ meets $x^2 + (y - 3)^2 = 29$.

2. Using an algebraic (and not geometric) method, determine the k such that the line $y = x + k$ **touches** the circle with equation $x^2 + y^2 = 1$.

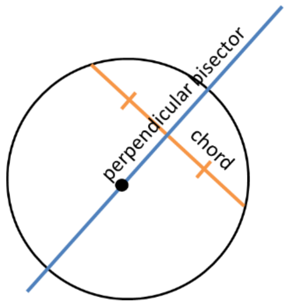
Exercise 6D Page 122

Tangents, chords and perpendicular bisectors

Reminder:



The tangent is perpendicular to the radius (at the point of intersection).



The perpendicular bisector of any chord passes through the centre of the circle.

Why are these useful?

Examples

1. The circle C has equation $(x-3)^2+(y-7)^2=100$.

a) Verify the point $P(11,1)$ lies on C.

b) Find an equation of the tangent to C at the point P, giving your answer in the form $ax+by+c=0$

2. A circle C has equation $(x-4)^2+(y+4)^2=10$. The line l is a tangent to the circle and has gradient -3. Find two possible equations for l, giving your answers in the form $y=mx+c$.

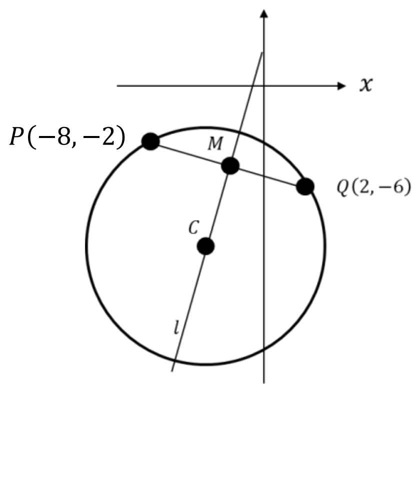
Finding the centre of a circle

Example:

The points P and Q lie on a circle with centre C, as shown in the diagram. The point P has coordinates $(-8,-2)$ and the point Q has coordinates $(2,-6)$. M is the midpoint of the line segment PQ.

The line l passes through the points M and C.

a) Find an equation for l.



b) Given that the y-coordinate of C is -9:
   i) show that the x-coordinate of C is -5.
   ii) find an equation of the circle.

Test Your Understanding

1. A circle has centre $C(3,5)$, and goes through the point $P(6,9)$. Find the equation of the tangent of the circle at the point P, giving your equation in the form $ax + by + c = 0$ where a, b, c are integers.

2. A circle passes through the points $A(0,0)$ and $B(4,2)$. The centre of the circle has x value -1. Determine the equation of the circle.

Extension

1. *MAT 2012 1A]* Which of the following lines is a tangent to the circle with equation

$x^2 + y^2 = 4$?

A) $x + y = 2$

B) $y = x - 2\sqrt{2}$

C) $x = \sqrt{2}$

D) $y = \sqrt{2} - x$

2. *[AEA 2006 Q4]* The line with equation $y = mx$ is a tangent to the circle C_1 with equation $(x + 4)^2 + (y - 7)^2 = 13$

(a) Show that m satisfies the equation $3m^2 + 56m + 36 = 0$

The tangents from the origin O to C_1 touch C_1 at the points A and B.

(b) Find the coordinates of the points A and B.

Another circle C_2 has equation $x^2 + y^2 = 13$. The tangents from the point $(4, -7)$ to C_2 touch it at the points P and Q.

(c) Find the coordinates of either the point P or the point Q.

3. *[STEP 2005 Q6]*

(i) The point A has coordinates $(5,16)$ and the point B has coordinates $(4, -4)$. The variable P has coordinates (x, y) and moves on a path such that $AP = 2BP$. Show that the Cartesian equation of the path of P is $(x + 7)^2 + y^2 = 100$.

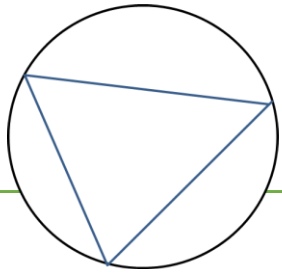
(ii) The point C has coordinates $(a, 0)$ and the point D has coordinates $(b, 0)$. The variable point Q moves on a path such that $QC = k \times QD$, where $k > 1$.

Exercise 6E Page 126

Given that the path of Q is the same as the path of P, show that

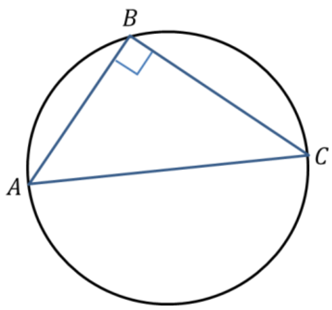
$$\frac{a+7}{b+7} = \frac{a^2+51}{b^2+51}$$

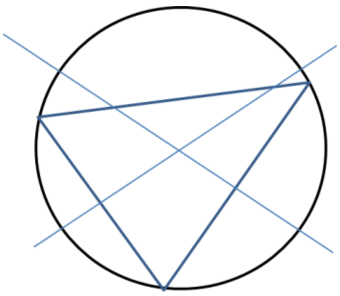
Show further that $(a+7)(b+7) = 100$, in the case $a \neq b$.



Triangles in Circles

- The triangle **inscribes** the circle.
  (A shape inscribes another if it is inside and its boundaries touch but do not intersect the outer shape)
- The circle **circumscribes** the triangle.
- If the circumscribing shape is a circle, it is known as the **circumcircle** of the triangle.
- The centre of a circumcircle is known as the **circumcentre**.





Examples

1. The points $A(-8,1)$, $B(4,5), C(-4,9)$ lie on a circle.

a) Show that AB is a diameter of the circle.

2. The points $A(0,2)$, $B(2,0), C(8,18)$ lie on the circumference of a circle. Determine the equation of the circle.

Extension

[STEP 2009 Q8 Edited] If equation of the circle C is $(x-2t)^2+(y-t)^2=t^2$, where t is a positive number, it can be shown that C touches the line $y=0$ as well as the line $3y=4x$.

Find the equation of the incircle of the triangle formed by the lines $y=0$, $3y=4x$ and $4y+3x=15$.

Note: The incircle of a triangle is the circle, lying totally inside the triangle, that touches all three sides.