Lower 6 Chapter 6

Circles

Chapter Overview

1. Perpendicular bisector recap

2. Equations of circles

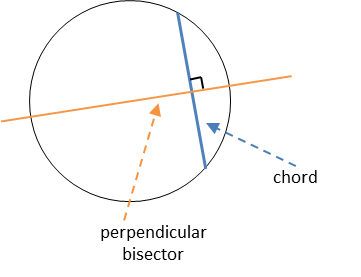
3. Intersections of lines and circles

4. Chords, tangents and perpendicular bisectors

5. Circumscribing Triangles



Perpendicular bisectors and mid-points



Example:

Find the equation of the perpendicular bisector of A (2,5) and B (6,7).

Test Your Understanding:

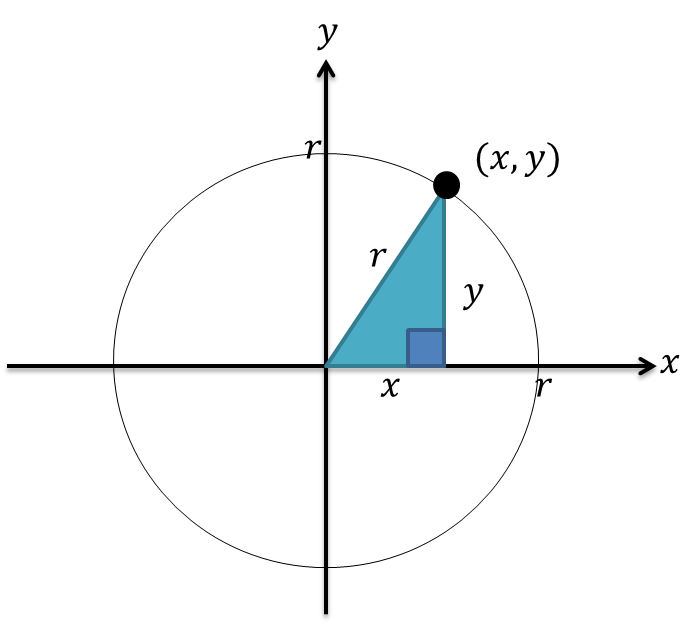
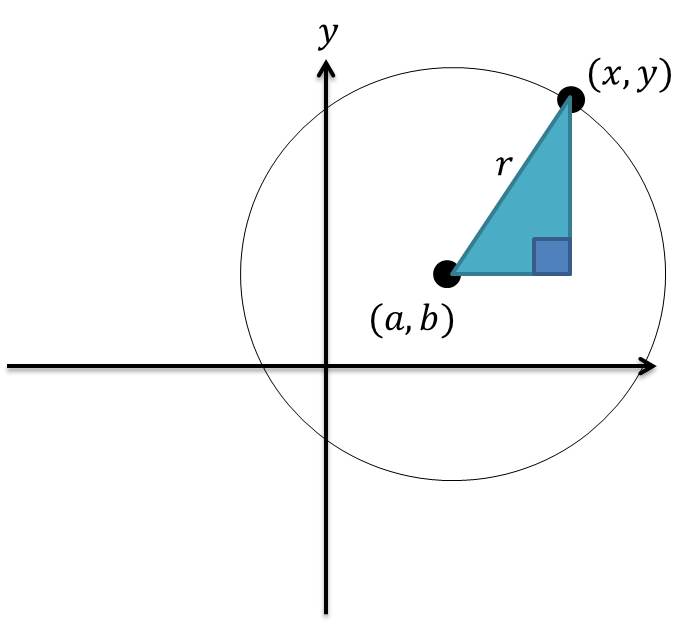
1. Find the perpendicular bisector of the line where and have the coordinates:



2. A line segment is the diameter of a circle with centre . If has coordinates , what are the coordinates of ?

Exercise 6A/B Page 115 - 117

Equation of a circle



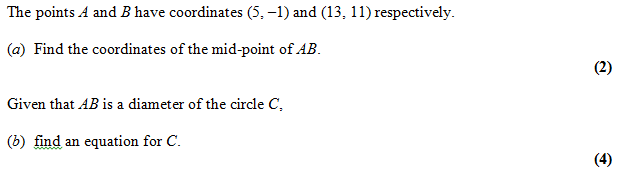
Examples:

1.

|  |  |  |
| --- | --- | --- |
| Centre | Radius | Equation |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

2. A line segment is the diameter of a circle, where and have coordinates and respectively. Determine the equation of the circle.

Test your understanding

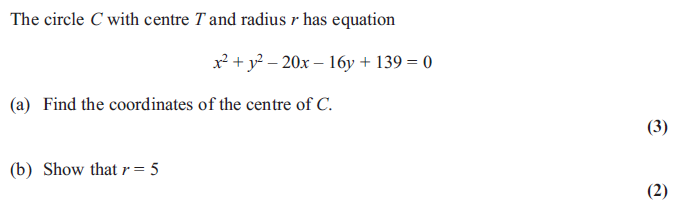


Completing the Square

Example

Find the centre and radius of the circle with equation

Test your understanding



Extension:

1. [MAT 2009 1B] The point on the circle which is closest to the origin, is at what distance from the origin?

2. [MAT 2007 1D]

The point on the circle which is closest to the circle

has what coordinates?

3. [MAT 2016 1I] Let and be positive real numbers. If then the largest that can equal is what?

Give your expression in terms of and .

Exercise 6C Page 119 - 120

The Intersection of Lines and Circles

Example: Show that the line never intersects the circle with equation

.

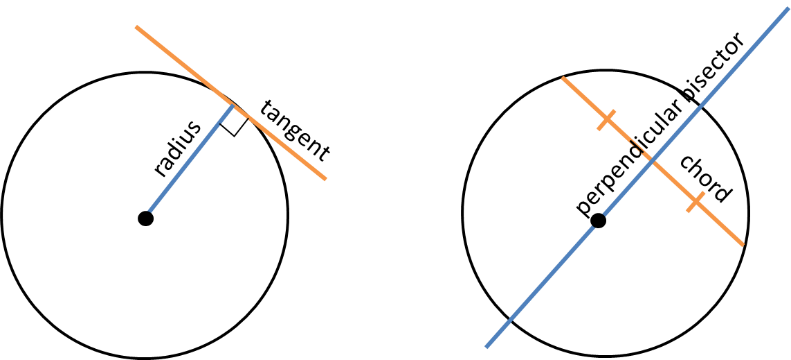
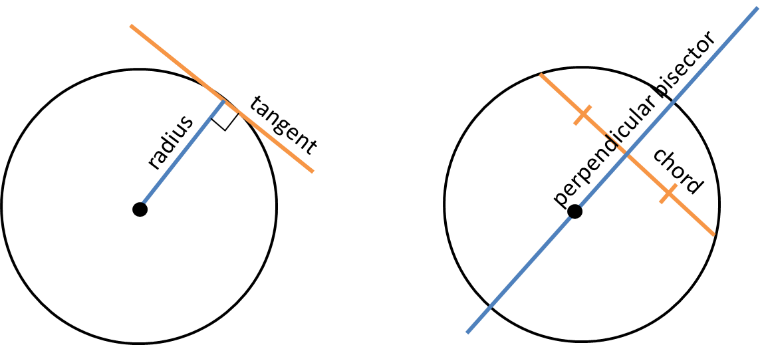
Test your understanding:

1. Find the points of intersection where the line meets .

2. Using an algebraic (and not geometric) method, determine the such that the line **touches** the circle with equation .

Exercise 6D Page 122

Tangents, chords and perpendicular bisectors

Reminder:

The perpendicular bisector of any chord passes through the centre of the circle.

The tangent is perpendicular to the radius (at the point of intersection).

Why are these useful?

Examples

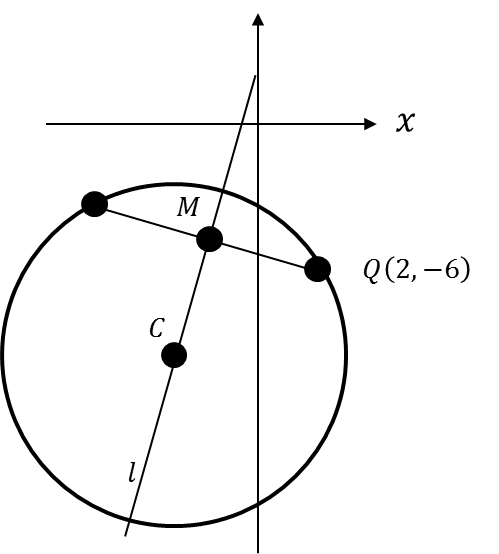
1. The circle has equation .

1. Verify the point lies on .
2. Find an equation of the tangent to at the point , giving your answer in the form

2. A circle has equation . The line is a tangent to the circle and has gradient -3. Find two possible equations for , giving your answers in the form .

Finding the centre of a circle

Example:

The points and lie on a circle with centre , as shown in the diagram. The point has coordinates and the point has coordinates . is the midpoint of the line segment .

The line passes through the points and .

a) Find an equation for .

b) Given that the -coordinate of is -9:  
 i) show that the -coordinate of is -5.  
 ii) find an equation of the circle.

Test Your Understanding

1. A circle has centre , and goes through the point . Find the equation of the tangent of the circle at the point , giving your equation in the form where are integers.

2. A circle passes through the points and . The centre of the circle has value -1. Determine the equation of the circle.

Extension

1. *MAT 2012 1A]* Which of the following lines is a tangent to the circle with equation

?



2. *[AEA 2006 Q4]* The line with equation is a tangent to the circle with equation

(a) Show that satisfies the equation

The tangents from the origin to touch at the points and .

(b) Find the coordinates of the points and .

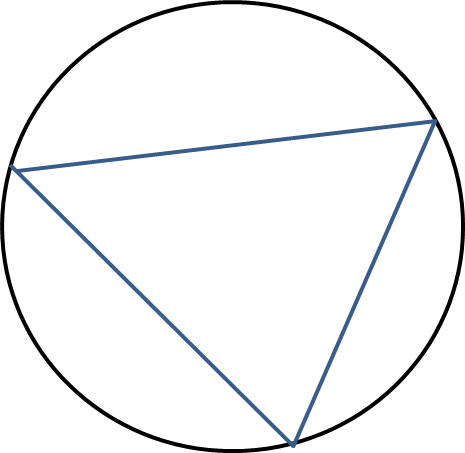
Another circle has equation . The tangents from the point to touch it at the points and .

(c) Find the coordinates of either the point or the point .

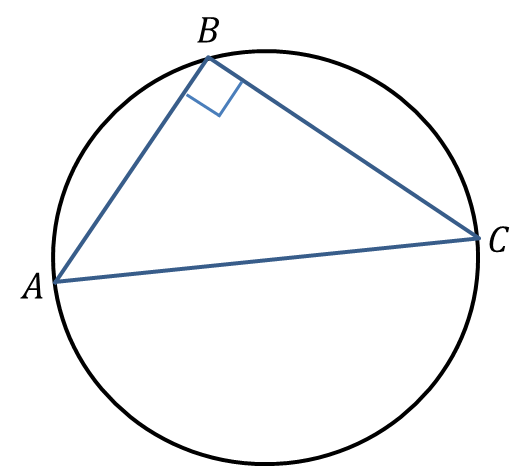
3. *[STEP 2005 Q6]*

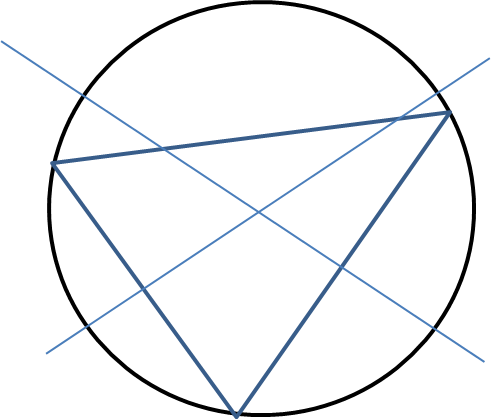
1. The point has coordinates and the point has coordinates . The variable has coordinates and moves on a path such that . Show that the Cartesian equation of the path of is .
2. The point has coordinates and the point has coordinates . The variable point moves on a path such that , where .  
   Given that the path of is the same as the path of , show that  
   Show further that , in the case .

Exercise 6E Page 126

Triangles in Circles

* The triangle **inscribes** the circle.  
  (A shape inscribes another if it is inside and its boundaries touch but do not intersect the outer shape)
* The circle **circumscribes** the triangle.
* If the circumscribing shape is a circle, it is known as the **circumcircle** of the triangle.
* The centre of a circumcircle is known as the **circumcentre**.





Examples

1. The points lie on a circle.

a) Show that is a diameter of the circle.

2. The points lie on the circumference of a circle. Determine the equation of the circle.

Extension

*[STEP 2009 Q8 Edited]* If equation of the circle is , where is a positive number, it can be shown that touches the line as well as the line .

Find the equation of the incircle of the triangle formed by the lines , and .

**Note**: The incircle of a triangle is the circle, lying totally inside the triangle, that touches all three sides.

Exercise 6F Page 131