

Lower 6 Chapter 5

Linear Graphs

Chapter Overview

1. $y = mx + c$
2. Parallel and perpendicular lines
3. Lengths and Areas
4. Modelling

3.1

Understand and use the equation of a straight line, including the forms

$$y - y_1 = m(x - x_1) \text{ and} \\ ax + by + c = 0;$$

Gradient conditions for two straight lines to be parallel or perpendicular.

Be able to use straight line models in a variety of contexts.

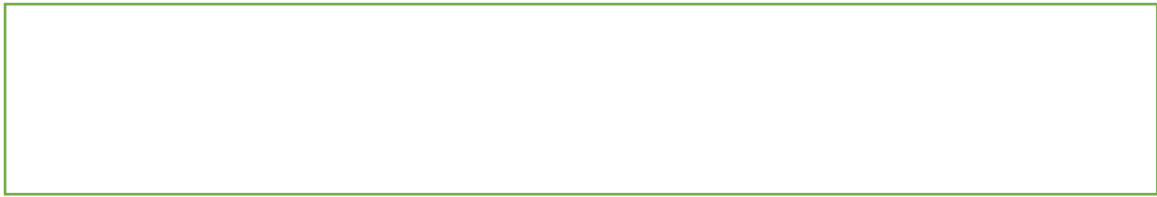
To include the equation of a line through two given points, and the equation of a line parallel (or perpendicular) to a given line through a given point.

$$m' = m \text{ for parallel lines and } m' = -\frac{1}{m}$$

for perpendicular lines

For example, the line for converting degrees Celsius to degrees Fahrenheit, distance against time for constant speed, etc.

1. Linear Graphs



Examples:

1. The point $(5, a)$ lies on the line with equation $y = 3x + 2$. Determine the value of a .

2. Find the coordinate of the point where the line $2x + y = 5$ cuts the x -axis.

Test Your Understanding:

Determine where the line $x + 2y = 3$ crosses both the axes

Gradient



Examples:

Find the gradient of the line between the following sets of points:

1. $(1, 4)$ $(3, 10)$

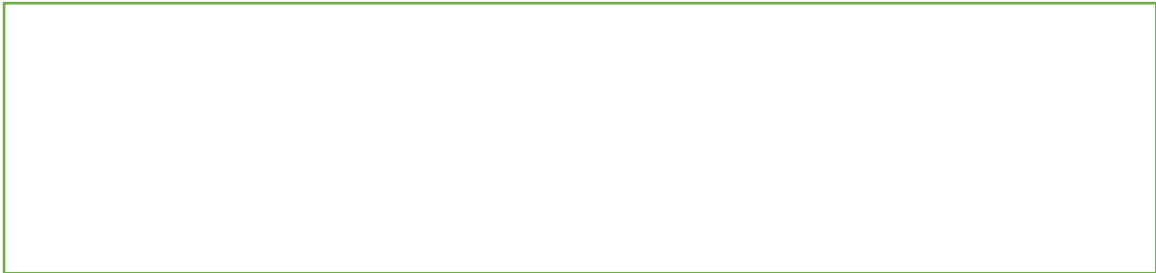
2. $(5, 7)$ $(8, 1)$

3. $(2, 2)$ $(-1, 10)$

4. Show that the points $A(3,4)$, $B(5,5)$, $C(11,8)$ all lie on a straight line.

5. The line joining $(2, -5)$ to $(4, a)$ has gradient -1 . Work out the value of a .

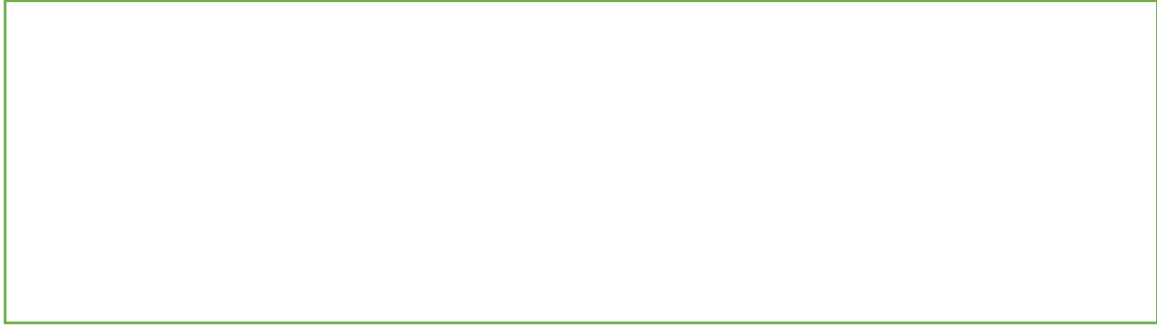
$$\underline{y = mx + c}$$



Example:

Determine the gradient and y -intercept of the line with equation $4x - 3y + 5 = 0$

$$\underline{ax + by + c = 0}$$



Example

Express $y = \frac{1}{3}x - \frac{2}{3}$ in the form $ax + by + c = 0$, where a, b, c are integers.

Test Your Understanding

Express $y = \frac{2}{5}x + \frac{3}{5}$ in the form $ax + by + c = 0$, where a, b, c are integers.

Equations using one point and the gradient

Example

Find the equation of the line that goes through (3,5) and has gradient 2.

Quickfire Questions

<u>Gradient</u>	<u>Point</u>	<u>(Unsimplified) Equation</u>
<u>3</u>	<u>(1,2)</u>	
<u>5</u>	<u>(3,0)</u>	
<u>2</u>	<u>(-3,4)</u>	
<u>$\frac{1}{2}$</u>	<u>(1,-5)</u>	
<u>9</u>	<u>(-4,-4)</u>	

Finding a line using 2 Points:



Example

1. Find the equation of the line that goes through $(4,5)$ and $(6,2)$, giving your equation in the form

$$ax + by + c = 0.$$

Test Your Understanding:

1. Find the equation of the line that goes through $(-1,9)$ and $(4,5)$, giving your equation in the form

$$ax + by + c = 0.$$

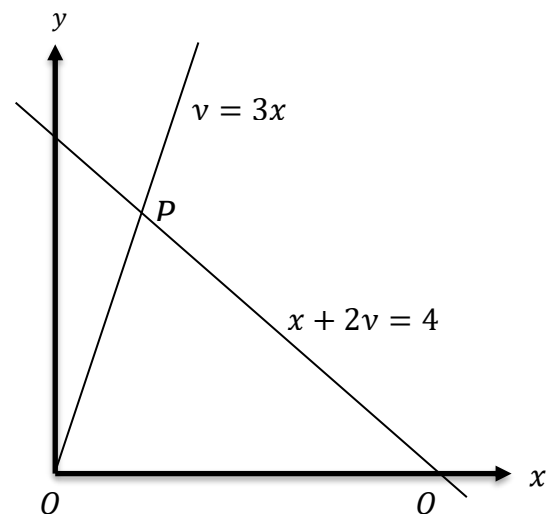
Intersection of Lines:



Example

The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point P .

- Determine the coordinates of P .
- The line $x + 2y = 4$ intersects the x -axis at the point Q . Determine the coordinate of Q .



Test Your Understanding

The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

(a) Find an equation for L_1 in the form $ax + by + c = 0$, where a , b and c are integers.

(4)

The line L_2 has equation $3y + 4x - 30 = 0$.

(b) Find the coordinates of the point of intersection of L_1 and L_2 .

(3)

Perpendicular Lines



Quickfire Questions

Gradient	Gradient of Perpendicular Line
2	
-3	
$\frac{1}{4}$	
5	
$-\frac{2}{7}$	
$\frac{7}{5}$	

Problems

1. A line is goes through the point (9,10) and is perpendicular to another line with equation $y = 3x + 2$. What is the equation of the line?

2. A line L_1 goes through the points $A(1,3)$ and $B(3,-1)$. A second line L_2 is perpendicular to L_1 and passes through point B. Where does L_2 cross the x-axis?

3. Are the following lines parallel, perpendicular, or neither?

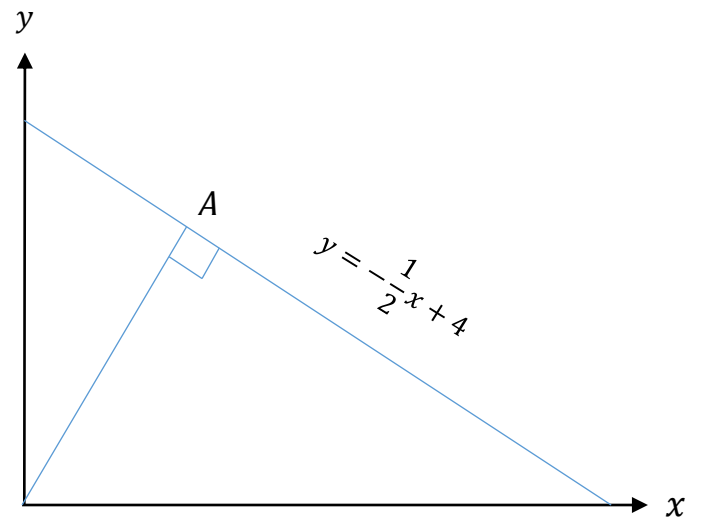
$$y = \frac{1}{2}x$$

$$2x - y + 4 = 0$$

Test Your Understanding

1. A line goes through the point $(4,7)$ and is perpendicular to another line with equation $y = 2x + 2$. What is the equation of the line? Put your answer in the form $ax + by + c = 0$, where a, b, c are integers.

2. Determine the point A .



Extension

1. [MAT 2004 1D]

What is the reflection of the point $(3,4)$ in the line $3x + 4y = 50$?

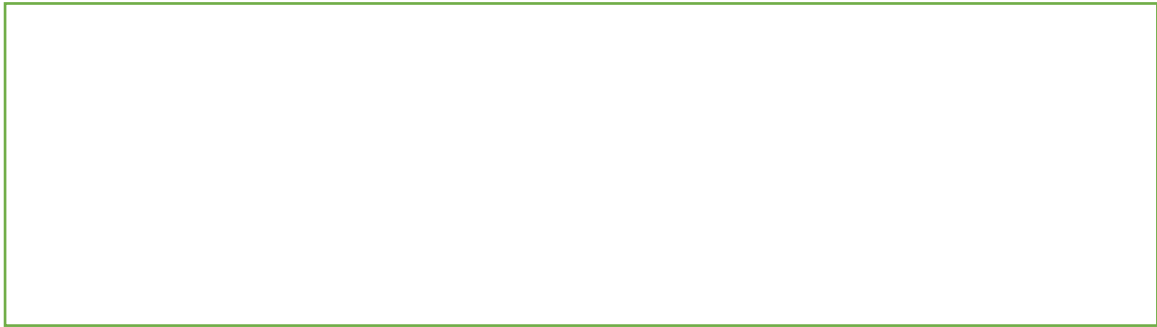
2. [MAT 2014 1D] The reflection of the point $(1,0)$ in the line $y = mx$ has coordinates: (in terms of m)

3. [STEP I 2004 Q6] The three points A, B, C have coordinates $(p_1, q_1), (p_2, q_2)$ and (p_3, q_3) , respectively. Find the point of intersection of the line joining A to the midpoint of BC , and the line joining B to the midpoint of AC . Verify that this point lies on the line joining C to the midpoint of AB .

The point H has coordinates $(p_1 + p_2 + p_3, q_1 + q_2 + q_3)$. Show that if the line AH intersects the line BC at right angles, then $p_2^2 + q_2^2 = p_3^2 + q_3^2$, and write down a similar result if the line BH intersects the line AC at right angles.

Deduce that if AH is perpendicular to BC and also BH is perpendicular to AC , then CH is perpendicular to AB .

Distances between points



Examples

Find the distance between

$(3,4)$ and $(5,7)$

$(5,1)$ and $(6,-3)$

$(0,-2)$ and $(-1,3)$

Test Your Understanding

Find the distance between:

$(1,10)$ and $(4,14)$

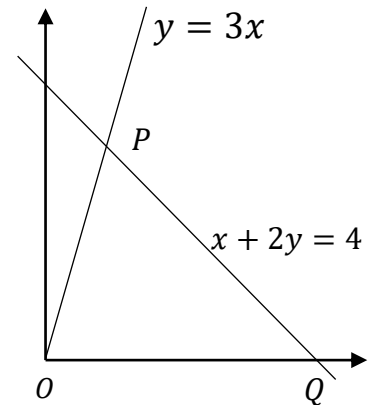
$(3,-1)$ and $(0,1)$

$(-4,-2)$ and $(-12,4)$

Area of Shapes

Example 1

The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point P .



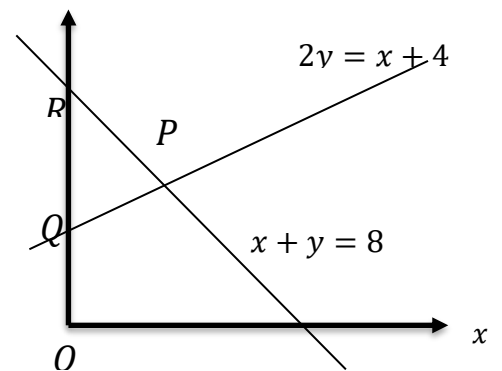
a) Determine the coordinates of P .

b) The line $x + 2y = 4$ intersects the x -axis at the point Q . Determine the area of the triangle OPQ .

When $y = 0$, $x = 4$

Example 2

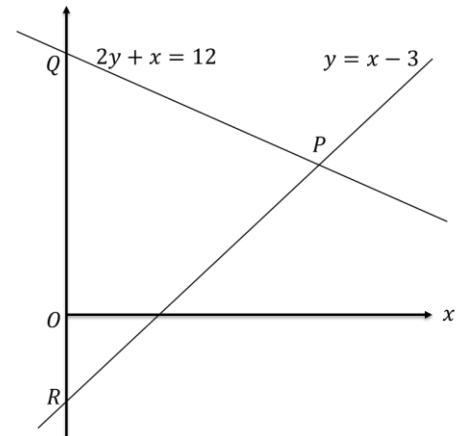
a) Determine the length of PQ



b) Determine the area PQR .

Test Your Understanding:

a) Determine the coordinate of P .



b) Determine the area of PQR .

c) Determine the length PQ .

Extension

[MAT 2001 1C]

The shortest distance from the origin to the line $3x + 4y = 25$ is what?

Modelling with Linear Graphs

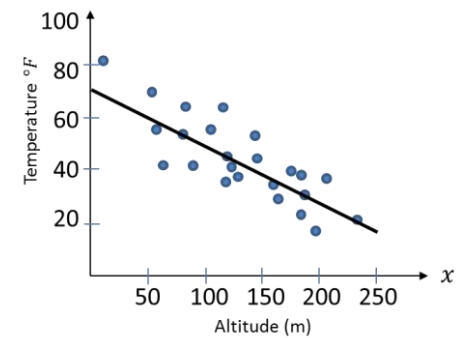
Many real life variables have a 'linear' relationship, i.e. there is a fixed increase/decrease in one variable each time the other variable goes up by 1 unit.

Example

The temperature y at different points on a mountain is recorded at different altitudes x .

Suppose we were to use a linear model $y = mx + c$.

a) Determine m and c (you can assume the line goes through $(0,70)$ and $(250,20)$).



b) Interpret the meaning of m and c in this context

c) Predict at what altitude the temperature reaches $0^{\circ}F$

Evaluating a Model



Example:

The current population of Bickerstonia is 26000. This year (2017) the population increased by 150. Matt decides to model the population P based on the years t after 2017 by the linear model:

$$P = mt + c$$

Why might this not be a suitable model?