Chapter 4

Graphs and Transformations

Chapter Overview

1. Polynomial Graphs

a. Cubic Graphs

b. Quartic Graphs

c. Reciprocal Graphs

2. Points of Intersection

3. Graph Transformations







Polynomial Graphs

Cubics

Examples

1. Sketch the curve with equation

We consider the shape, the roots and the y – intercept.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Equation | If | Resulting Shape | As  As  If | Resulting Shape |
| As  As |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

2. Sketch the curve with equation

3. Sketch the curve with equation

4. Sketch the curve with equation

5. Sketch the curve with equation

Finding the equation: example

The graph shows a sketch of the curve with equation . The curve passes through the point (–1, 0) and touches the -axis at the point (2, 0). The curve has a maximum at the point (0, 4). The equation of the curve can be written in the form where , and are integers.

Calculate the values of .



Test Your Understanding:

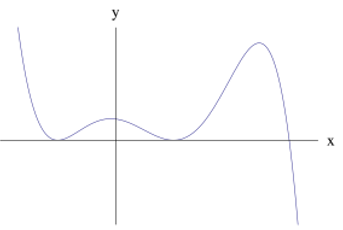
1. Sketch the curve with equation

2. Sketch the curve with equation

3. A curve has this shape , touches the axis at 3 and crosses the axis at -2. Give a suitable equation for this graph.

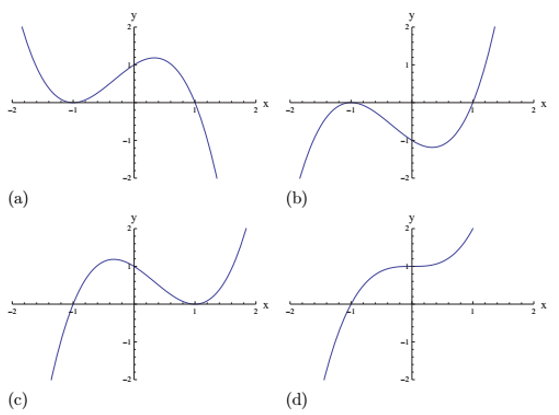
4. Extension. Sketch the curve with equation

[MAT 2012 1E] Which one of the following equations could possibly have the graph given below?





[MAT 2011 1A] A sketch of the graph appears on which of the following axis?



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Quartics:

Examples:

1. Sketch the curve with equation

2. Sketch the curve with equation

3. Sketch the curve with equation

4. Sketch the curve with equation

Test Your Understanding

1. Sketch the curve with equation

2. Sketch the curve with equation

Extension:

*[STEP I 2012 Q2a]*

1. Sketch
2. For what values of does the equation have the following number of distinct roots (i) 0, (ii) 1, (iii) 2, (iv) 3, (v) 4.

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Reciprocal Graphs

1. Sketch 2. Sketch

3. Sketch 4. Sketch

5. On the same axes, sketch and

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Points of Intersection

If and , then the values of the points of intersection can be found when .

Examples:

1. On the same diagram sketch the curves with equations and . Find the coordinates of their points of intersection.

2. On the same diagram sketch the curves with equations and , where are positive constants. State, giving a reason, the number of real solutions to the equation

Test Your Understanding

On the same diagram sketch the curves with equations and , and hence find the coordinates of any points of intersection.

Extension

1. [MAT 2005 1B]

The equation

1. has as a solution;
2. has no real solutions;
3. has an odd number of real solutions;
4. has twenty real solutions.

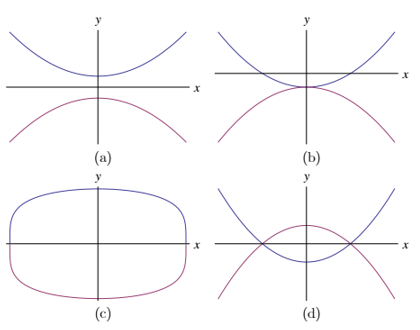
2. [MAT 2010 1A] The values of for which the line intersects the parabola are precisely

1. B)

C) or D)

3. [MAT 2013 1D]

Which of the following sketches is a graph of ?



Exercise 4D Page 69

Transformations of Graphs

It is important to understand the effects of simple transformations on the graph .

:

|  |  |
| --- | --- |
| Function | Effect |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
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We can think of it like this:

|  |  |  |
| --- | --- | --- |
|  | Affects which axis? | What we expect or opposite? |
| Change **inside** |  |  |
| Change **outside** |  |  |

Examples: Describe the transformation

Example

1. Sketch
2. Sketch
3. Sketch . On the same axes, sketch , where .
4. Sketch . On the same axes, sketch the graph with equation .

Reflections

Example

If , sketch and on the same axes.

Test your understanding

1. If , sketch and on the same axes.
2. Sketch the graph of , ensuring you indicate any intercepts with the axes.

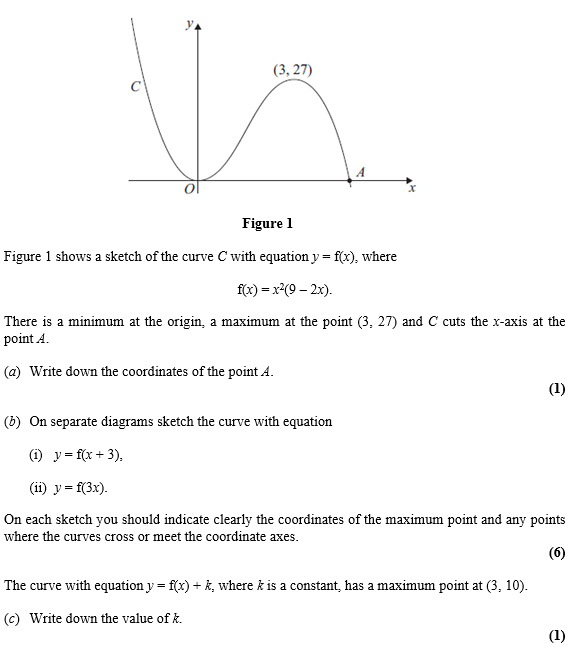
Exercise 4E/F Page 74

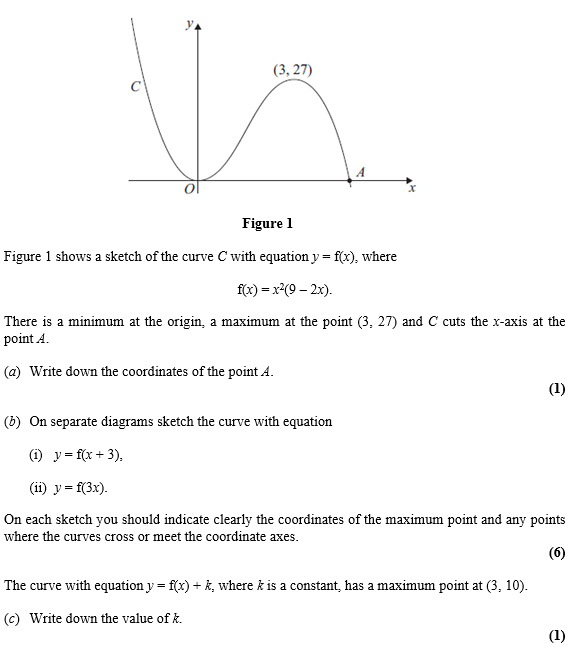
The effect of transformations on specific points

Sometimes you will not be given the original function, but will be given a sketch with specific points and features you need to transform.

Where would each of these points end up?

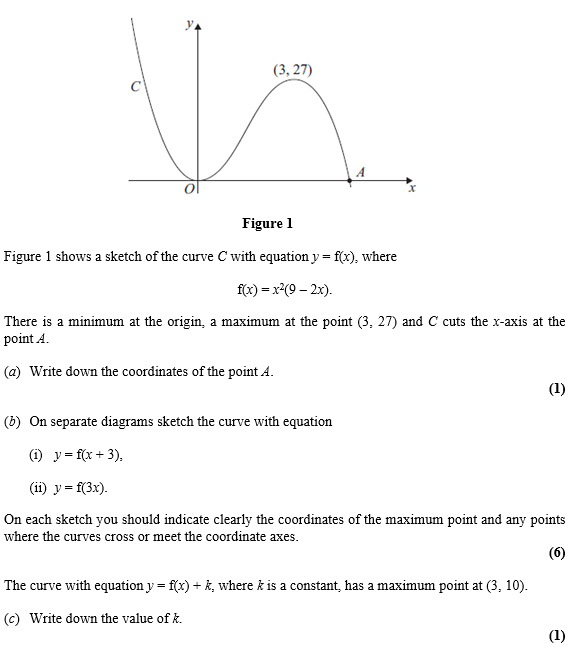
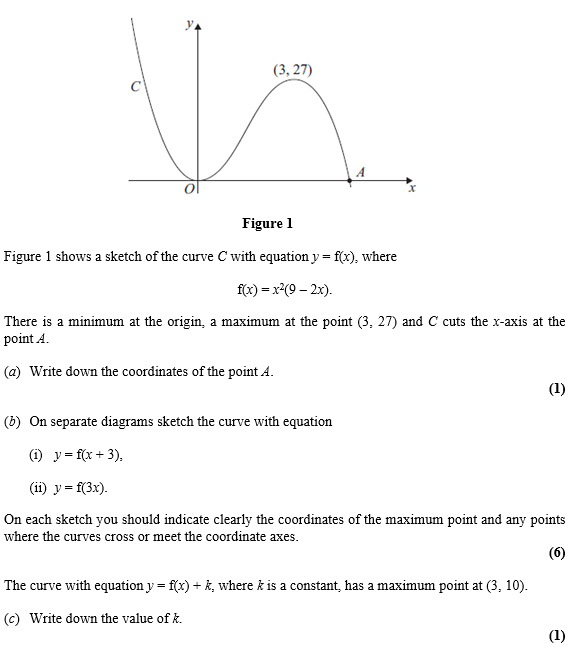
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Test Your Understanding



a)

b)

c)

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