Chapter 3

Equations and Inequalities

Chapter Overview

1. Simultaneous Equations

2. Simultaneous Equations Using Graphs

3. Set Builder Notation

4. Solving Inequalities

5. Sketching Inequalities

2.4	**Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation.**	**The quadratic may involve powers of 2 in one unknown or in both unknowns,** **e.g. solve** $y = 2x + 3$, $y = x^2 - 4x + 8$ **or** $2x - 3y = 6$, $x^2 - y^2 + 3x = 50$

2.5	**Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically,**	**e.g. solving** $ax + b > cx + d$, $px^2 + qx + r \geq 0$, $px^2 + qx + r < ax + b$ **and interpreting the third inequality as the range of** x **for which the curve** $y = px^2 + qx + r$ **is below the line with equation** $y = ax + b$
	including inequalities with brackets and fractions.	**These would be reducible to linear or quadratic inequalities** **e.g.** $\frac{a}{x} < b$ **becomes** $ax < bx^2$
	Express solutions through correct use of 'and' and 'or', or through set notation.	**So, e.g.** $x < a$ **or** $x > b$ **is equivalent to** $\{x : x < a\} \cup \{x : x > b\}$ **and** $\{x : c < x\} \cap \{x : x < d\}$ **is equivalent to** $x > c$ **and** $x < d$
	Represent linear and quadratic inequalities such as $y > x + 1$ **and** $y > ax^2 + bx + c$ **graphically.**	**Shading and use of dotted and solid line convention is required.**

Simultaneous Equations

Linear Equations:

Example:

Solve the simultaneous equations

$$3x + y = 8$$

$$2x - 3y = 9$$

Method 1 : Elimination

Method 2: Substitution

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Linear and Quadratic

Example:

Solve the simultaneous equations:

$$x + 2y = 3$$

$$x^2 + 3xy = 10$$

Test Your Understanding:

1. Solve the simultaneous equations: $3x^2 + y^2 = 21$ and $y = x + 1$

Extension:

1.

[MAT 2012 1G] There are *positive* real numbers x and y which solve the equations $2x + ky = 4, x + y = k$ for:

A) All values of k;
B) No values of k;
C) $k = 2$ only;
D) Only $k > -2$

2. *[STEP 2010 Q1]* Given that

$$5x^2 + 2y^2 - 6xy + 4x - 4y \equiv a(x - y + 2)^2 + b(cx + y)^2 + d$$

a) Find the values of a, b, c, d.

b) Solve the simultaneous equations:

$$5x^2 + 2y^2 - 6xy + 4x - 4y = 9$$
$$6x^2 + 3y^2 - 8xy + 8x - 8y = 14$$

(Hint: Can we use the same method in (a) to rewrite the second equation?)

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Simultaneous Equations and Graphs

Examples:

1a. On the same axes, draw the graphs of $2x + y = 3$ and

$y = x^2 - 3x + 1$

1b. Use your graph to write down the solutions to the simultaneous equations

1c. What algebraic method could we have used to show the graphs would have intersected twice?

Example 2

a) On the same axes, draw the graphs of:

$$y = 2x - 2 \qquad y = x^2 + 4x + 1$$

b) Prove algebraically that the lines never meet

Question: The line with equation $y = 2x + 1$ meets the curve with equation $kx^2 + 2y + (k - 2) = 0$ at exactly one point. Given that k is a positive constant:

a) Find the value of k.

b) For this value of k, find the coordinates of this point of intersection

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<u>Set Builder Notation</u>

Recap from GCSE:

- We use curly braces to list the values in a set, e.g. $A = \{1,4,6,7\}$
- If A and B are sets then $A \cap B$ is the **intersection** of A and B, giving a set which has the elements in A **<u>and</u>** B.
- $A \cup B$ is the **union** of A and B, giving a set which has the elements in A **<u>or</u>** in B.
- $\emptyset$ is the empty set, i.e. the set with nothing in it.
- Sets can also be infinitely large. $\mathbb{N}$ is the set of natural numbers (all positive integers), $\mathbb{Z}$ is the set of all integers (including negative numbers and 0) and $\mathbb{R}$ is the set of all real numbers (including all possible decimals).
- We write $x \in A$ to mean "x <u>is a member of</u> the set A". So $x \in \mathbb{R}$

Examples:

1. $\{2x : x \in \mathbb{Z}\}$

2. $\{2^x : x \in \mathbb{N}\}$

3. $\{xy\text{: } x, y\ are\ prime\}$

Solving Inequalities

Linear inequalities Examples

1. $2x + 1 > 5$

2. $3(x - 5) \geq 5 - 2(x - 8)$

3. $-x \geq 2$

Combining Inequalities

When combining inequalities always draw a number line to help!

Example:

If $x < 3$ and $2 \leq x < 4$, what is the combined solution set?

Quadratic Inequalities:

Examples

1. Solve $x^2 + 2x - 15 > 0$

2. Solve $x^2 + 2x - 15 \leq 0$

3. Solve $x^2 + 5x \geq -4$

4. Solve $x^2 < 9$

Test Your Understanding

Find the set of values of x for which

(*a*) $3(x - 2) < 8 - 2x$, **(2)**

(*b*) $(2x - 7)(1 + x) < 0$, **(3)**

(*c*) both $3(x - 2) < 8 - 2x$ **and** $(2x - 7)(1 + x) < 0$. **(1)**

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(*a*) show that $q^2 + 8q < 0$. **(2)**

(*b*) Hence find the set of possible values of q. **(3)**

Division by x

Find the set of values for which $\frac{6}{x} > 2, \quad x \neq 0$

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Sketching Inequalities:

Examples

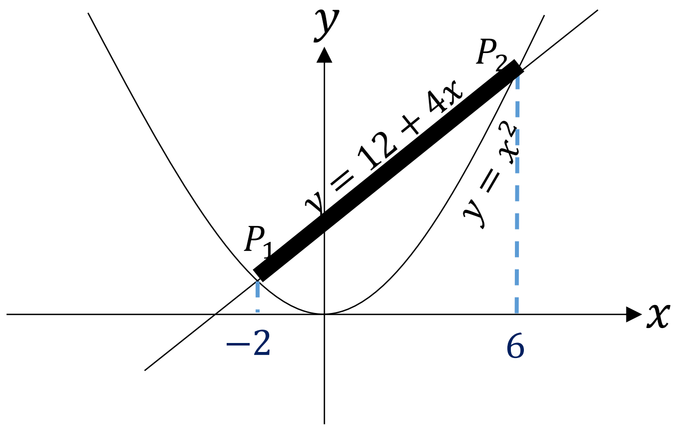
1. L_1 has equation $y = 12 + 4x$. L_2 has equation $y = x^2$.

The diagram shows a sketch of L_1 and L_2 on the same axes.

a) Find the coordinates of P_1 and P_2, the points of intersection.

b) Hence write down the solution to the inequality

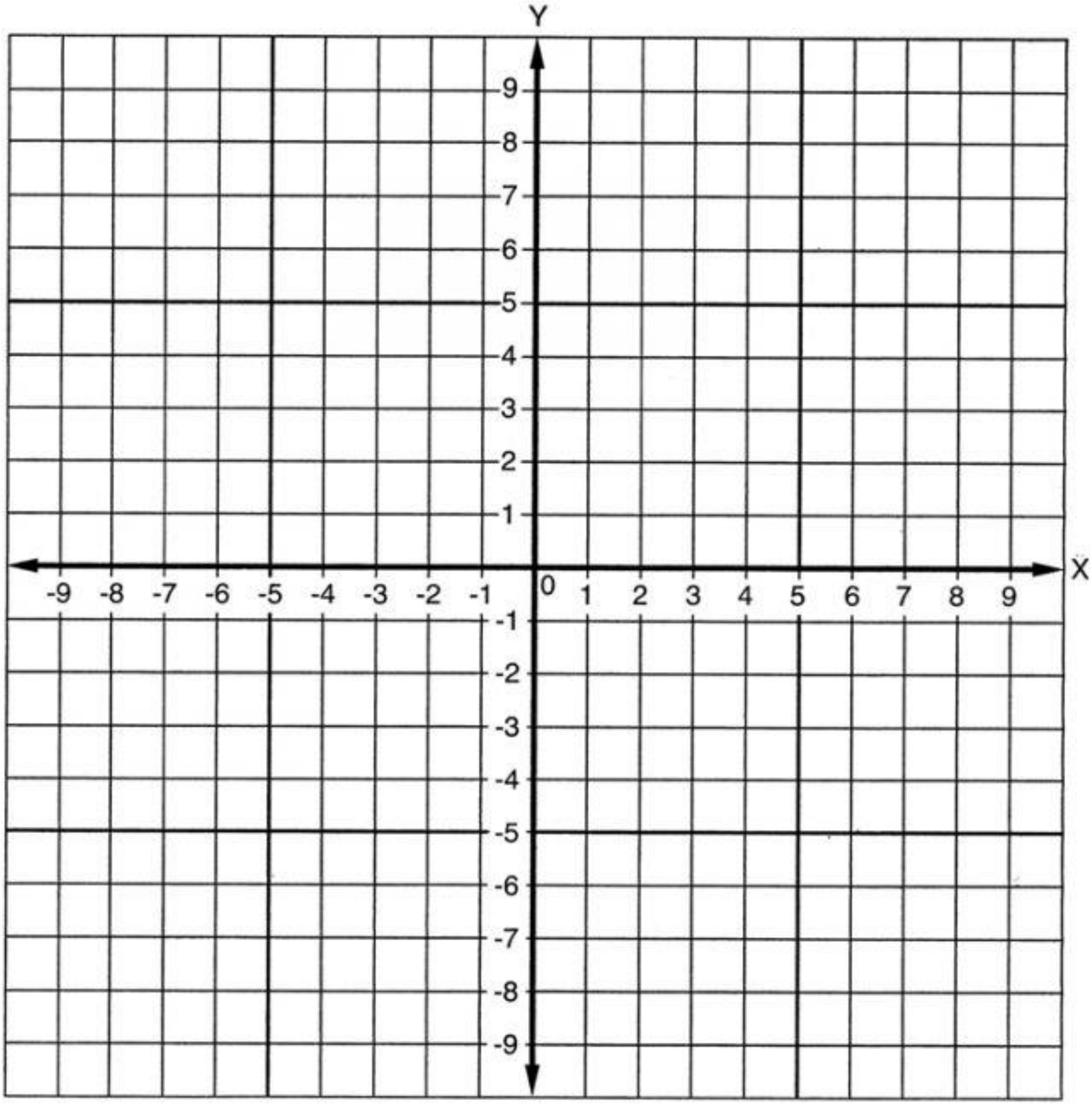
$12 + 4x > x^2$.



2. Shade the region that satisfies the inequalities:

$$2y + x < 14$$

$$y \geq x^2 - 3x - 4$$



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