Lower 6 Chapter 10

Trigonometric Identities and Equations

Chapter Overview

1. Know exact trig values for $30°, 45°, 60°$ and understand unit circle.
2. Use identities $\frac{\sin(x)}{\cos(x)}≡\tan(x)$ and
$sin^{2}x+cos^{2}x≡1$
3. Solve equations of the form $\sin(\left(nθ\right))=k$ and $\sin(\left(θ\pm α\right)=k)$
4. Solve equations which are quadratic in sin/cos/tan.



sin/cos/tan of $30°, 45°, 60°$

You will frequently encounter angles of $30°, 60°, 45°$ in geometric problems. Why?

Although you will always have a calculator, you need to know how to derive these.

**All you need to remember:**

** Draw half a unit square and half an equilateral triangle of side 2.**

$$\sin(\left(30°\right))=$$

$$\cos(\left(30°\right))=$$

$$\tan(\left(30°\right))=$$

$$\sin(\left(60°\right))=$$

$$\cos(\left(60°\right))=$$

$$\tan(\left(60°\right))=$$

$$\sin(\left(45°\right))=$$

$$\cos(\left(45°\right))=$$

$$\tan(\left(45°\right))=$$

The Unit Circle and Trigonometry

For values of $θ$ in the range $0<θ<90°$, you know that $\sin(θ)$ and $\cos(θ)$ are lengths on a right-angled triangle:

And what would be the **gradient** of the bold line (hypotenuse)?

But how do we get the rest of the graph for $sin, cos$ and $tan$ when $90°\leq θ\leq 360°$?

The point $P$ on a unit circle, such that $OP$ makes an angle $θ$ with the positive $x$-axis, has coordinates $\left(\cos(θ),\sin(θ)\right)$.
$OP $has gradient $\tan(θ)$.

Angles are always measured **anticlockwise**.

We can consider the coordinate P $\left(\cos(θ),\sin(θ)\right)$ as $θ$ increases from 0 to $360°$…

Use the unit circle to determine each value in the table, **using either “0”, “+ve”, “-ve”, “1”, “-1” or “undefined”**.



The unit circle explains the behaviour of the trigonometric graphs beyond $90°$.

However, the easiest way to remember whether $\sin(\left(x\right)),\cos(\left(x\right)),\tan(\left(x\right))$ are positive or negative is to just do a **very quick sketch (preferably mentally!)** of the corresponding graph.

**Note:** The textbook uses something called ‘*CAST diagrams*’. We will not be using them in this booklet, but you may wish to look at this technique as an alternative approach to various problems in the chapter.

A Few Trigonometric Angle Laws

The following are all easily derivable using a quick sketch of a trigonometric graph, and are merely a convenience so you don’t always have to draw out a graph every time.

You are highly encouraged to **memorise these** so that you can do exam questions faster.



Examples

Without a calculator, work out the value of each below:





Exercise 10A and B Pg 207 and 209

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**Trigonometric identities**

$$tan θ=\frac{sin θ}{cos θ}$$

$$sin^{2}θ+cos^{2}θ=1$$

Examples

Prove that $1-\tan(θ)\sin(θ)\cos(θ)≡cos^{2}θ$

Prove that $\tan(θ)+\frac{1}{\tan(θ)}≡\frac{1}{\sin(θ)\cos(θ)}$

Simplify $5-5sin^{2}θ$

Test your understanding

Prove that $\frac{\tan(x)\cos(x)}{\sqrt{1-cos^{2}x}}≡1$

Prove that $\frac{cos^{4}θ-sin^{4}θ}{cos^{2}θ}≡1-tan^{2}θ$

Prove that $tan^{2}θ≡\frac{1}{cos^{2}θ}-1$

Exercise 10C Pg 211/212

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**Solving trigonometric equations**

Solve $\sin(θ)=\frac{1}{2}$ in the interval $0\leq θ\leq 360°$.

Solve $5\tan(θ)=10$ in the interval $-180°\leq θ<180°$

Solve $\sin(θ)=-\frac{1}{2}$ in the interval $0\leq θ\leq 360°$.

Solve $\sin(θ)=\sqrt{3}\cos(θ)$ in the interval $0\leq θ\leq 360°$.

Solve $2\cos(θ)=\sqrt{3}$ in the interval $0\leq θ\leq 360°$.

Solve $\sqrt{3}\sin(θ)=\cos(θ)$ in the interval $-180°\leq θ\leq 180°$.

Exercise 10D Pg 215/216

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**Harder equations**

Solve $\cos(3x=-\frac{1}{2})$ in the interval $0\leq x\leq 360°$.

Solve $\sin((2x+30°)=\frac{1}{\sqrt{2}})$ in the interval $0\leq x\leq 360°$.

Solve $\sin(x)=2\cos(x)$ in the interval $0\leq x<300°$



Exercise 10E Pg 218/219

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**Quadratics in sin/cos/tan**

Solve $5sin^{2}x+3\sin(x)-2=0$ in the interval $0\leq x\leq 360°$.

Solve $tan^{2}θ=4$ in the interval $0\leq x\leq 360°$.

Solve $2cos^{2}x+9\sin(x)=3sin^{2}x$ in the interval $-180°\leq x\leq 180°$.



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