Chapter 1

Algebraic Expressions

Chapter Overview

1. Basic Index Laws
2. Negative/ Fractional Indices
3. Factorise Quadratics and Cubics
4. Expanding Brackets
5. Surds

Topics	What students need to learn:		
	Content		Guidance
2 Algebra and functions	2.1	**Understand and use the laws of indices for all rational exponents.**	$a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$ **The equivalence of** $a^{\frac{m}{n}}$ **and** $\sqrt[n]{a^m}$ **should be known.**
	2.2	**Use and manipulate surds, including rationalising the denominator.**	**Students should be able to simplify algebraic surds using the results** $\left(\sqrt{x}\right)^2 = x$, $\sqrt{xy} = \sqrt{x}\sqrt{y}$ **and** $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right) = x - y$

Basic Index Laws

Examples

1. Simplify $(a^3)^2 \times 2a^2$

2. Simplify $(4x^3y)^3$

3. Simplify $2x^2(3 + 5x) - x(4 - x^2)$

4. Simplify $\frac{x^3-2x}{3x^2}$ (2 methods)

Test Your Understanding:

1. Simplify $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$

2. Simplify $\frac{2x+x^5}{4x^3}$

3.Expand and simplify $2x(3-x^2)-4x^3(3-x)$

4.Simplify $2^x \times 3^x$

Extension

[MAT 2006 1A]

Which of the following numbers is largest?

- $\left((2^3)^2\right)^3$
- $(2^3)^{(2^3)}$
- $2^{\left((3^2)^3\right)}$
- $2^{\left(3^{(2^3)}\right)}$

[MAT 2012 1B]

Let $N = 2^k \times 4^m \times 8^n$ where k, m, n are positive whole numbers.

Then N will definitely be a square number whenever:

- k is even;
- $k+n$ is odd;
- k is odd but $m+n$ is even;
- $k+n$ is even.

Negative and Fractional Indices

1. Prove that $x^{\frac{1}{2}} = \sqrt{x}$

2. Evaluate $27^{-\frac{1}{3}}$

3. Evaluate $32^{\frac{2}{5}}$

4. Simplify $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

5. Evaluate $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

6. If $b = \frac{1}{9}a^2$, determine $3b^{-2}$ in the form ka^n where k, n are constants

Extension

[MAT 2007 1A]

Let r and s be integers. Then

$$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$

is an integer if

- $r + s \leq 0$
- $s \leq 0$
- $r \leq 0$
- $r \geq s$

Exercise 1D Page 11

Brackets: Expanding

Example: $(x+1)(x+2)(x+3)$

Questions

1. Expand and simplify

$$(x+5)(x-2)(x+1)$$

2. Expand and simplify:

$$2(x-3)(x-4)$$

3. Expand and simplify:

$$(2x-1)^3$$

Extension

[MAT 2002 1B]

Of the following three alleged algebraic identities, at least one is wrong.

(i) $yz(z-y)+zx(x-z)+xy(y-x)$
$=(z-y)(x-z)(y-x)$
(ii) $yz(z-y)+zx(x-z)+xy(y-x)$
$=(z-y)(z-x)(y-x)$
(iii) $yz(x+y)+zx(z+x)+xy(y+x)$
$=(z+y)(z+x)(y+x)$

Which of the following statements are correct? Tick all that apply.

- [ ] (i)
- [ ] (ii)
- [ ] (iii)

[MAT 2007 1E]

If x and n are integers then

$$(1-x)^n(2-x)^{2n}(3-x)^{3n}(4-x)^{4n}(5-x)^{5n}$$

is:

- negative when $n>5$ and $x<5$
- negative when n is odd and $x>5$
- negative when n is a multiple of 3 and $x>5$
- negative when n is even and $x<5$

Brackets: Factorising

Examples:

1. $x^2 - 5x - 14$

2. $2x^2 + 5x - 12$

3. $4x^2 - 9$

4. $x^3 - x$

5. $x^3 + 3x^2 + 2x$

Test your understanding: Factorise completely

1. $6x^2 + x - 2$

2. $x^3 - 7x^2 + 12x$

3. $x^4 - 1$

4. $x^3 - 1$

Surds:

1. $\sqrt{3} \times 2$

2. $3\sqrt{5} \times 2\sqrt{5}$

3. $\sqrt{8}$

4. $\sqrt{12} + \sqrt{27}$

5. $(\sqrt{8} + 1)(\sqrt{2} - 3)$

Extension:

[SMC 2014 Q24] Which of the following is smallest?

- $10 - 3\sqrt{11}$
- $8 - 3\sqrt{7}$
- $5 - 2\sqrt{6}$
- $9 - 4\sqrt{5}$
- $7 - 4\sqrt{3}$

[SMC 2012 Q21] Which of the following numbers does *not* have a square root in the form $x + y\sqrt{2}$, where x and y are positive integers?

- $17 + 12\sqrt{2}$
- $22 + 12\sqrt{2}$
- $38 + 12\sqrt{2}$
- $54 + 12\sqrt{2}$
- $73 + 12\sqrt{2}$

Rationalising the denominator:

Examples:

1. $\frac{3}{\sqrt{2}}$

2. $\frac{6}{\sqrt{3}}$

3. $\frac{7}{\sqrt{7}}$

4. $\frac{15}{\sqrt{5}} + \sqrt{5}$

Test your understanding:

$\frac{12}{\sqrt{3}}$

$\frac{2}{\sqrt{6}}$

$\frac{4\sqrt{2}}{\sqrt{8}}$

More Complicated Examples:

1. $\frac{3}{\sqrt{6}-2}$

2. $\frac{4}{\sqrt{3}+1}$

3. $\frac{3\sqrt{2}+4}{5\sqrt{2}-7}$

Test Your Understanding: Rationalise the denominator and simplify

1. $\frac{4}{\sqrt{5}-2}$

2. $\frac{2\sqrt{3}-1}{3\sqrt{3}+1}$

3. Solve $y\left(\sqrt{3}-1\right)=8$

Give your answer in the form $a+b\sqrt{3}$ where a and b are integers.