

# Chapter 1

## Algebraic Expressions

### Chapter Overview

1. Basic Index Laws
2. Negative/ Fractional Indices
3. Factorise Quadratics and Cubics
4. Expanding Brackets
5. Surds

Topics	What students need to learn:		
	Content	Guidance	
2 Algebra and functions	2.1	<p><b>Understand and use the laws of indices for all rational exponents.</b></p>	<p><math>a^m \times a^n = a^{m+n}</math>, <math>a^m \div a^n = a^{m-n}</math>, <math>(a^m)^n = a^{mn}</math></p> <p>The equivalence of <math>a^{\frac{m}{n}}</math> and <math>\sqrt[n]{a^m}</math> should be known.</p>
	2.2	<p><b>Use and manipulate surds, including rationalising the denominator.</b></p>	<p>Students should be able to simplify algebraic surds using the results</p> <p><math>(\sqrt{x})^2 = x</math>, <math>\sqrt{xy} = \sqrt{x}\sqrt{y}</math> and</p> <p><math>(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = x - y</math></p>

## Basic Index Laws



### Examples

1. Simplify  $(a^3)^2 \times 2a^2$

2. Simplify  $(4x^3y)^3$

3. Simplify  $2x^2(3 + 5x) - x(4 - x^2)$

4. Simplify  $\frac{x^3 - 2x}{3x^2}$  ( 2 methods)

### Test Your Understanding:

1. Simplify  $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$

2. Simplify  $\frac{2x+x^5}{4x^3}$

3. Expand and simplify  $2x(3 - x^2) - 4x^3(3 - x)$

4. Simplify  $2^x \times 3^x$

### Extension

[MAT 2006 1A]

Which of the following numbers is largest?

- $\left((2^3)^2\right)^3$
- $(2^3)^{(2^3)}$
- $2\left((3^2)^3\right)$
- $2\left(3^{(2^3)}\right)$

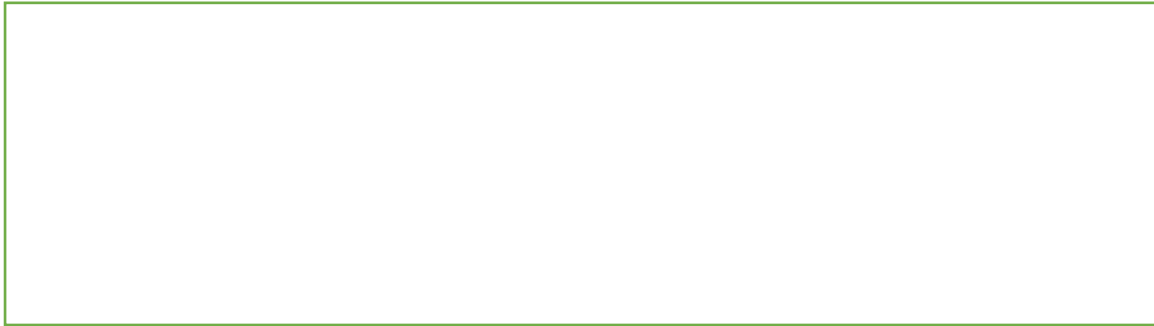
[MAT 2012 1B]

Let  $N = 2^k \times 4^m \times 8^n$  where  $k, m, n$  are positive whole numbers.

Then  $N$  will definitely be a square number whenever:

- $k$  is even;
- $k + n$  is odd;
- $k$  is odd but  $m + n$  is even;
- $k + n$  is even.

## Negative and Fractional Indices



1. Prove that  $x^{\frac{1}{2}} = \sqrt{x}$

2. Evaluate  $27^{-\frac{1}{3}}$

3. Evaluate  $32^{\frac{2}{5}}$

4. Simplify  $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

5. Evaluate  $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

6. If  $b = \frac{1}{9}a^2$ , determine  $3b^{-2}$  in the form  $ka^n$  where  $k, n$  are constants

## Extension

[MAT 2007 1A]

Let  $r$  and  $s$  be integers. Then

$$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$

is an integer if

- $r + s \leq 0$
- $s \leq 0$
- $r \leq 0$
- $r \geq s$