Chapter 1

Algebraic Expressions

Chapter Overview

1. Basic Index Laws
2. Negative/ Fractional Indices
3. Factorise Quadratics and Cubics
4. Expanding Brackets
5. Surds

Topics	What students need to learn:		
	Content		Guidance
2 Algebra and functions	2.1	**Understand and use the laws of indices for all rational exponents.**	$a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$ **The equivalence of** $a^{\frac{m}{n}}$ **and** $\sqrt[n]{a^m}$ **should be known.**
	2.2	**Use and manipulate surds, including rationalising the denominator.**	**Students should be able to simplify algebraic surds using the results** $\left(\sqrt{x}\right)^2 = x$, $\sqrt{xy} = \sqrt{x}\sqrt{y}$ **and** $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right) = x - y$

Basic Index Laws

Examples

1. Simplify $(a^3)^2 \times 2a^2$

2. Simplify $(4x^3y)^3$

3. Simplify $2x^2(3 + 5x) - x(4 - x^2)$

4. Simplify $\frac{x^3-2x}{3x^2}$ (2 methods)

Test Your Understanding:

1. Simplify $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$

2. Simplify $\frac{2x+x^5}{4x^3}$

3.Expand and simplify $2x(3-x^2)-4x^3(3-x)$

4.Simplify $2^x \times 3^x$

Extension

[MAT 2006 1A]

Which of the following numbers is largest?

- $\left((2^3)^2\right)^3$
- $(2^3)^{(2^3)}$
- $2^{\left((3^2)^3\right)}$
- $2^{\left(3^{(2^3)}\right)}$

[MAT 2012 1B]

Let $N = 2^k \times 4^m \times 8^n$ where k, m, n are positive whole numbers.

Then N will definitely be a square number whenever:

- k is even;
- $k+n$ is odd;
- k is odd but $m+n$ is even;
- $k+n$ is even.

Negative and Fractional Indices

1. Prove that $x^{\frac{1}{2}} = \sqrt{x}$

2. Evaluate $27^{-\frac{1}{3}}$

3. Evaluate $32^{\frac{2}{5}}$

4. Simplify $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

5. Evaluate $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

6. If $b = \frac{1}{9}a^2$, determine $3b^{-2}$ in the form ka^n where k, n are constants

Extension

[MAT 2007 1A]

Let r and s be integers. Then

$$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$

is an integer if

- $r + s \leq 0$
- $s \leq 0$
- $r \leq 0$
- $r \geq s$

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