The Factor Theorem

Examples

1. Show that $(x-2)$ is a factor of x^3+x^2-4x-4.

2. Fully factorise $2x^3+x^2-18x-9$.

Using the factor theorem to find unknown coefficients:

1. Given that $2x + 1$ is a factor of $6x^3 + ax^2 + 1$, determine the value of a.

Test your understanding

$$f(x) = 6x^3 + 13x^2 - 4$$

(a) ~~Use the remainder theorem to find the remainder when $f(x)$ is divided by $(2x + 3)$.~~ **(2)**

(b) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$. **(2)**

(c) Factorise $f(x)$ completely. **(4)**

2. Given that $3x - 1$ is a factor of $3x^3 + 11x^2 + ax + 1$, determine the value of a.

Extension

1. *[MAT 2006 1E]* The cubic $x^3 + ax + b$ has both $(x - 1)$ and $(x - 2)$ has factors. Determine the values of a and b.

2. [MAT 2009 1I] The polynomial $n^2x^{2n+3} - 25nx^{n+1} + 150x^7$ has $x^2 - 1$ as a factor

A) for no values of n;

B) for $n = 10$ only;

C) for $n = 15$ only;

D) for $n = 10$ and $n = 15$ only.

The **remainder theorem** states that if $f(x)$ is divided by $(x - a)$, the remainder is $f(a)$. This similarly works whenever a makes the divisor 0.

(No longer required for A Level)

3. [MAT 2013 1G] Let $n \geq 2$ be an integer and $p_n(x)$ be the polynomial

$$p_n(x) = (x - 1) + (x - 2) + \cdots + (x - n)$$

What is the remainder, in terms of n, when $p_n(x)$ is divided by $p_{n-1}(x)$?