Lower 6 Chapter 7

Algebraic Methods

Chapter Overview

- 1. Algebraic Fractions
- 2. Algebraic Long Division
- 3. Factor Theorem
- 4. Proof

Algebra and functions continued	2.6	Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem. Simplify rational expressions, including by factorising and cancelling, and algebraic division (by linear expressions only).	Only division by $(ax + b)$ or $(ax - b)$ will be required. Students should know that if $f(x) = 0$ when $x = a$, then $(x - a)$ is a factor of $f(x)$. Students may be required to factorise cubic expressions such as $x^3 + 3x^2 - 4$ and $6x^3 + 11x^2 - x - 6$. Denominators of rational expressions will be linear or quadratic, e.g. $\frac{1}{ax + b}$, $\frac{ax + b}{px^2 + qx + r}$, $\frac{x^3 + a^3}{x^2 - a^2}$
1 Proof	1.1	Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including: Proof by deduction	Examples of proofs: Proof by deduction e.g. using completion of the square, prove that $n^2 - 6n + 10$ is positive for all values of n or, for example, differentiation from first principles for small positive integer powers of x or proving results for arithmetic and geometric series. This is the most commonly used method of proof throughout this specification
		Proof by exhaustion	Proof by exhaustion This involves trying all the options. Suppose x and y are odd integers less than 7. Prove that their sum is divisible by 2.
		Disproof by counter example	Disproof by counter example e.g. show that the statement " $n^2 - n + 1$ is a prime number for all values of n'' is untrue

Simplifying Algebraic Fractions

Examples

$$1. \frac{x^2 - 1}{x^2 + x}$$

$$2. \, \frac{x^2 + 3x + 2}{x + 1}$$

$$3.\,\frac{2x^2+11x+12}{x^2+9x+20}$$

$$4. \frac{4 - x^2}{x^2 + 2x - 8}$$