Transformations of Graphs

It is important to understand the effects of simple transformations on the graph $y=f\left( x\right)$.

$For y=f\left( x\right)$:

|  |  |
| --- | --- |
| Function | Effect |
| $$f\left( x+a\right)$$ |  |
| $$f\left(x-a\right)$$ |  |
| $$f\left( x\right)+a$$ |  |
| $$f\left( x\right)-a$$ |  |
| $$f\left( ax\right)$$ |  |
| $$af\left( x\right)$$ |  |
| $$f\left( -x\right)$$ |  |
| $$-f\left( x\right)$$ |  |

We can think of it like this:

|  |  |  |
| --- | --- | --- |
|  | Affects which axis? | What we expect or opposite? |
| Change **inside** $f( )$ |  |  |
| Change **outside** $f( )$ |  |  |

Examples: Describe the transformation

1. $y=f\left(x-3\right)$
2. $y=f\left(x\right)+4$
3. $y=f\left(5x\right)$
4. $y=2f\left(x\right)$

Example

1. Sketch $y=x^{2}+3$
2. Sketch $y=\frac{2}{x+1}$
3. Sketch $y=x(x+2)$. On the same axes, sketch $y=\left(x-a\right)\left(x-a+2\right)$, where $a>2$.
4. Sketch $y=x^{2}\left(x-4\right)$. On the same axes, sketch the graph with equation $y=\left(2x\right)^{2}\left(2x-4\right)$.

Reflections

Example

If $y=x(x+2)$, sketch $y=f(x)$ and $y=-f(x)$ on the same axes.

Test your understanding

1. If $y=(x+1)(x-2)$, sketch $y=f(x)$ and $y=f(\frac{x}{3})$ on the same axes.
2. Sketch the graph of $y=\frac{2}{x}+1$, ensuring you indicate any intercepts with the axes.

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