

## Points of Intersection

If  $y = f(x)$  and  $y = g(x)$ , then the  $x$  values of the points of intersection can be found when  $f(x) = g(x)$ .

Examples:

1. On the same diagram sketch the curves with equations  $y = x(x - 3)$  and  $y = x^2(1 - x)$ . Find the coordinates of their points of intersection.

2. On the same diagram sketch the curves with equations  $y = x^2(3x - a)$  and  $y = \frac{b}{x}$ , where  $a, b$  are positive constants. State, giving a reason, the number of real solutions to the equation  $x^2(3x - a) - \frac{b}{x} = 0$

## Test Your Understanding

On the same diagram sketch the curves with equations  $y = x(x - 4)$  and  $y = x(x - 2)^2$ , and hence find the coordinates of any points of intersection.

## Extension

1. [MAT 2005 1B]

The equation  $(x^2 + 1)^{10} = 2x - x^2 - 2$

- A) has  $x = 2$  as a solution;
- B) has no real solutions;
- C) has an odd number of real solutions;
- D) has twenty real solutions.

2. [MAT 2010 1A] The values of  $k$  for which the line  $y = kx$  intersects the parabola  $y = (x - 1)^2$  are precisely

- A)  $k \leq 0$                       B)  $k \geq -4$   
 C)  $k \geq 0$  or  $k \leq -4$     D)  $-4 \leq k \leq 0$

3. [MAT 2013 1D]

Which of the following sketches is a graph of  $x^4 - y^2 = 2y + 1$ ?

