Points of Intersection

If $y=f(x)$ and $y=g(x)$, then the $x$ values of the points of intersection can be found when $f\left(x\right)=g(x)$.

Examples:

1. On the same diagram sketch the curves with equations $y=x(x-3)$ and $y=x^{2}\left(1-x\right)$. Find the coordinates of their points of intersection.

2. On the same diagram sketch the curves with equations $y=x^{2}\left(3x-a\right)$ and $y=\frac{b}{x}$, where $a,b$ are positive constants. State, giving a reason, the number of real solutions to the equation $x^{2}\left(3x-a\right)-\frac{b}{x}=0$

Test Your Understanding

On the same diagram sketch the curves with equations $y=x(x-4)$ and $y=x\left(x-2\right)^{2}$, and hence find the coordinates of any points of intersection.

Extension

1. [MAT 2005 1B]

The equation $\left(x^{2}+1\right)^{10}=2x-x^{2}-2$

1. has $x=2$ as a solution;
2. has no real solutions;
3. has an odd number of real solutions;
4. has twenty real solutions.

2. [MAT 2010 1A] The values of $k$ for which the line $y=kx$ intersects the parabola $y=\left(x-1\right)^{2}$ are precisely

1. $k\leq 0$ B) $k\geq -4$

C) $k\geq 0$ or $k\leq -4$ D) $-4\leq k\leq 0$

3. [MAT 2013 1D]

Which of the following sketches is a graph of $x^{4}-y^{2}=2y+1$?



Exercise 4D Page 69