Chapter 4

Graphs and Transformations

Chapter Overview

1. Polynomial Graphs
    a. Cubic Graphs
    b. Quartic Graphs
    c. Reciprocal Graphs
2. Points of Intersection
3. Graph Transformations

Understand and use graphs of functions; sketch curves defined by simple equations including polynomials	**Graph to include simple cubic and quartic functions,** **e.g. sketch the graph with equation $y = x^2(2x-1)^2$**
$y = \frac{a}{x}$ and $y = \frac{a}{x^2}$ **(including their vertical and horizontal asymptotes)** **Interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations.**	**The asymptotes will be parallel to the axes e.g. the asymptotes of the curve with equation $y = \frac{2}{x+a} + b$ are the lines with equations $y = b$ and $x = -a$**

Understand the effect of simple transformations on the graph of $y = \mathrm{f}(x)$, including sketching associated graphs: $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$ and combinations of these transformations	Students should be able to find the graphs of $y = \lvert \mathrm{f}(x) \rvert$ and $y = \lvert \mathrm{f}(-x) \rvert$, given the graph of $y = \mathrm{f}(x)$. Students should be able to apply a combination of these transformations to any of the functions in the A Level specification (quadratics, cubics, quartics, reciprocal, $\frac{a}{x^2}$, $\lvert x \rvert$, $\sin x$, $\cos x$, $\tan x$, e^x and a^x) and sketch the resulting graph. Given the graph of $y = \mathrm{f}(x)$, students should be able to sketch the graph of, e.g. $y = 2\mathrm{f}(3x)$, or $y = \mathrm{f}(-x) + 1$, and should be able to sketch (for example) $y = 3 + \sin 2x$, $y = -\cos\left(x + \frac{\pi}{4}\right)$

Polynomial Graphs

Equation	If $a > 0$	Resulting Shape	If $a < 0$	Resulting Shape
$y = ax^2 + bx + c$	As $x \to \infty$, $y \to \infty$ As $x \to -\infty$, $y \to \infty$		As $x \to \infty$, $y \to -\infty$ As $x \to -\infty$, $y \to -\infty$	
$y = ax^3 + bx^2 + cx + d$				
$y = ax^4 + bx^3 + cx^2 + dx + e$				
$y = ax^5 + bx^4 + \cdots$				

Cubics

Examples

1. Sketch the curve with equation $y = (x - 2)(1 - x)(1 + x)$

We consider the shape, the roots and the y – intercept.

2. Sketch the curve with equation $y = x^2(x - 1)$

3. Sketch the curve with equation $y = (2 - x)(x + 1)^2$

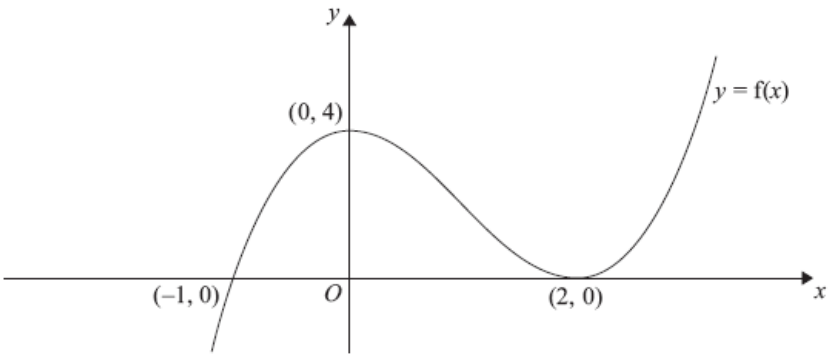
4. Sketch the curve with equation $y = (x - 4)^3$

5. Sketch the curve with equation $y = (x + 1)(x^2 + x + 1)$

Finding the equation: example

The graph shows a sketch of the curve C with equation $y = f(x)$. The curve C passes through the point (–1, 0) and touches the x-axis at the point (2, 0). The curve C has a maximum at the point (0, 4). The equation of the curve C can be written in the form $y = x^3 + ax^2 + bx + c$ where a, b and c are integers.

Calculate the values of a, b, c.

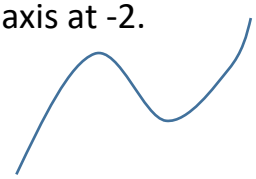


Test Your Understanding:

1. Sketch the curve with equation $y = x(x-3)^2$

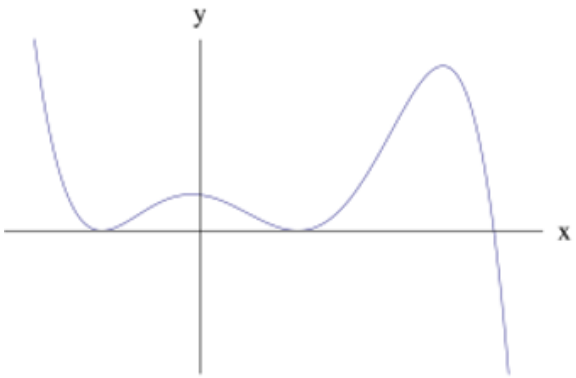
2. Sketch the curve with equation $y = -(x+2)^3$

3. A curve has this shape , touches the x axis at 3 and crosses the x axis at -2. Give a suitable equation for this graph.



4. Extension. Sketch the curve with equation $y = 2x^2(x-1)(x+1)^3$

[MAT 2012 1E] Which one of the following equations could possibly have the graph given below?



A) $y = (3 - x)^2(3 + x)^2(1 - x)$
B) $y = -x^2(x - 9)(x^2 - 3)$
C) $y = (x - 6)(x - 2)^2(x + 2)^2$
D) $y = (x^2 - 1)^2(3 - x)$

[MAT 2011 1A] A sketch of the graph $y = x^3 - x^2 - x + 1$ appears on which of the following axis?