Finding Equations of Tangents

Example

Find the equation of the **tangent** to the curve $y = x^2$ when $x = 3$.

Find the equation of the **normal** to the curve $y = x^2$ when $x = 3$.

Test your Understanding

Find the equation of the **normal** to the curve $y = x + 3\sqrt{x}$ when $x = 9$.

Extension

1. [STEP I 2005 Q2]

The point P has coordinates $(p^2, 2p)$ and the point Q has coordinates $(q^2, 2q)$, where p and q are non-zero and $p \neq q$. The curve C is given by $y^2 = 4x$. The point R is the intersection of the tangent to C at P and the tangent to C at Q. Show that R has coordinates $(pq, p + q)$.

The point S is the intersection of the normal to C at P and the normal to C at Q. If p and q are such that $(1,0)$ lies on the line PQ, show that S has coordinates $(p^2 + q^2 + 1, p + q)$, and that the quadrilateral $PSQR$ is a rectangle.

2. *STEP I 2012 Q4]*

The curve C has equation $xy = \frac{1}{2}$.

The tangents to C at the distinct points $P\left(p, \frac{1}{2p}\right)$ and $Q\left(q, \frac{1}{2q}\right)$, where p and q are positive, intersect at T and the normal to C at these points intersect at N. Show that T is the point

$$\left(\frac{2pq}{p+q}, \frac{1}{p+q}\right)$$

In the case $pq = \frac{1}{2}$, find the coordinates of N. Show (in this case) that T and N lie on the line $y = x$ and are such that the product of their distances from the origin is constant.

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