Finding Equations of Tangents

Example

Find the equation of the **tangent** to the curve $y=x^{2}$ when $x=3$.

Find the equation of the **normal** to the curve $y=x^{2}$ when $x=3$.

Test your Understanding

Find the equation of the **normal** to the curve $y=x+3\sqrt{x}$ when $x=9$.

Extension

*1. [STEP I 2005 Q2]*

The point $P$ has coordinates $\left(p^{2},2p\right)$ and the point $Q$ has coordinates $\left(q^{2},2q\right)$, where $p$ and $q$ are non-zero and $p\ne q$. The curve $C$ is given by $y^{2}=4x$. The point $R$ is the intersection of the tangent to $C$ at $P$ and the tangent to $C$ at $Q$. Show that $R$ has coordinates $\left(pq,p+q\right)$.

The point $S$ is the intersection of the normal to $C$ at $P$ and the normal to $C$ at $Q$. If $p$ and $q$ are such that $\left(1,0\right)$ lies on the line $PQ$, show that $S$ has coordinates $\left(p^{2}+q^{2}+1,p+q\right)$, and that the quadrilateral $PSQR$ is a rectangle.

2. *STEP I 2012 Q4]*

The curve $C$ has equation $xy=\frac{1}{2}$.

The tangents to $C$ at the distinct points $P\left(p,\frac{1}{2p}\right)$ and $Q\left(q,\frac{1}{2q}\right)$, where $p$ and $q$ are positive, intersect at $T$ and the normal to $C$ at these points intersect at $N$. Show that $T$ is the point

$$\left(\frac{2pq}{p+q},\frac{1}{p+q}\right)$$

In the case $pq=\frac{1}{2}$, find the coordinates of $N$. Show (in this case) that $T$ and $N$ lie on the line $y=x$ and are such that the product of their distances from the origin is constant.

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