Lower 6 Chapter 12

Differentiation

Chapter Overview

1. First Principles and finding the derivative of polynomials.
2. Find equations of tangents and normal to curves.
3. Identify increasing and decreasing functions.
4. Find and understand the second derivative $\frac{d^2y}{dx^2}$ or $f''(x)$
5. Find stationary points and determine their nature.
6. Sketch a gradient function.
7. Model real-life problems.

7 **Differentiation**	7.1	**Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change**	**Know that $\frac{dy}{dx}$ is the rate of change of y with respect to x.** **The notation $f'(x)$ may be used for the first derivative and $f''(x)$ may be used for the second derivative.**
		sketching the gradient function for a given curve	**Given for example the graph of $y = f(x)$, sketch the graph of $y = f'(x)$ using given axes and scale. This could relate speed and acceleration for example.**
		second derivatives	
		differentiation from first principles for small positive integer powers of x and for $\sin x$ and $\cos x$	**For example, students should be able to use, for $n = 2$ and $n = 3$, the gradient expression** $$\lim_{h \to 0}\left(\frac{(x+h)^n - x^n}{h}\right)$$ **Students may use δx or h**

Topics	What students need to learn:		
	Content		Guidance
7 **Differentiation** *continued*	7.1 *cont.*	**Understand and use the second derivative as the rate of change of gradient;** connection to convex and concave sections of curves and points of inflection.	Use the condition $f''(x) > 0$ implies a minimum and $f''(x) < 0$ implies a maximum for points where $f'(x) = 0$ Know that at an inflection point $f''(x)$ changes sign. Consider cases where $f''(x) = 0$ and $f'(x) = 0$ where the point may be a minimum, a maximum or a point of inflection (e.g. $y = x^n, n > 2$)
	7.2	**Differentiate x^n, for rational values of n, and related constant multiples, sums and differences.**	**For example, the ability to differentiate expressions such as** $(2x + 5)(x - 1)$ **and** $\frac{x^2 + 3x - 5}{4x^{\frac{1}{2}}}, x > 0,$ **is expected.**
		Differentiate e^{kx} and a^{kx}, $\sin kx$, $\cos kx$, $\tan kx$ and related sums, differences and constant multiples. Understand and use the derivative of $\ln x$	Knowledge and use of the result $\frac{d}{dx}(a^{kx}) = ka^{kx}\ln a$ is expected.
	7.3	**Apply differentiation to find gradients, tangents and normals**	**Use of differentiation to find equations of tangents and normals at specific points on a curve.**
		maxima and minima and stationary points. points of inflection	**To include applications to curve sketching. Maxima and minima problems may be set in the context of a practical problem.**
		Identify where functions are increasing or decreasing.	**To include applications to curve sketching.**

The Gradient Function

Example

The point A with coordinates $(4,16)$ lies on the curve with equation $y = x^2$.

At point A the curve has gradient g.

a) Show that $g = \lim_{h \to 0}(8 + h)$

b) Deduce the value of g.

Example

Prove from first principles that the derivative of $x^3 - 2x = 3x^2 - 2$

Test you understanding

Prove **from first principles** that the derivative of x^4 is $4x^3$.

Ex 12A/B page261