Quadratics in sin/cos/tan

Solve $5 \sin^2 x + 3 \sin x - 2 = 0$ in the interval $0 \leq x \leq 360°$.

Solve $\tan^2 \theta = 4$ in the interval $0 \leq x \leq 360°$.

Solve $2\cos^2 x + 9\sin x = 3\sin^2 x$ in the interval $-180° \leq x \leq 180°$.

(a) Show that the equation

$$5\sin x = 1 + 2\cos^2 x$$

can be written in the form

$$2\sin^2 x + 5\sin x - 3 = 0.$$

(2)

(b) Solve, for $0 \leq x < 360°$,

$$2\sin^2 x + 5\sin x - 3 = 0.$$

(4)

Extension

1 *[MAT 2010 1C]* In the range $0 \leq x < 360°$, the equation
$$\sin^2 x + 3 \sin x \cos x + 2 \cos^2 x = 0$$
Has how many solutions?

2 *[MAT 2014 1E]* As x varies over the real numbers, the largest value taken by the function
$$(4 \sin^2 x + 4 \cos x + 1)^2 \text{ equals}$$
what?