Trigonometric identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Examples

Prove that $1 - \tan \theta \sin \theta \cos \theta \equiv \cos^2 \theta$

Prove that $\tan \theta + \dfrac{1}{\tan \theta} \equiv \dfrac{1}{\sin \theta \cos \theta}$

Simplify $5 - 5 \sin^2 \theta$

Prove that $\dfrac{\tan x \cos x}{\sqrt{1-\cos^2 x}} \equiv 1$

Prove that $\dfrac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} \equiv 1 - \tan^2 \theta$

Prove that $\tan^2 \theta \equiv \dfrac{1}{\cos^2 \theta} - 1$

Extension:

[MAT 2008 1C] The simultaneous equations in x, y,

$$(\cos \theta)x - (\sin \theta)y = 2$$
$$(\cos \theta)x + (\sin \theta)y = 1$$

are solvable:

A) for all values of θ in range $0 \le \theta < 2\pi$

B) except for one value of θ in range $0 \le \theta < 2\pi$

C) except for two values of θ in range $0 \le \theta < 2\pi$

D) except for three values of θ in range $0 \le \theta < 2\pi$