Lower 6 Chapter 10

Trigonometric Identities and Equations

Chapter Overview

1. Know exact trig values for $30°,\ 45°,\ 60°$ and understand unit circle.
2. Use identities $\frac{\sin x}{\cos x} \equiv \tan x$ and $\sin^2 x + \cos^2 x \equiv 1$
3. Solve equations of the form $\sin(n\theta) = k$ and $\sin(\theta \pm \alpha) = k$
4. Solve equations which are quadratic in sin/cos/tan.

Trigonometry	5.3	Understand and use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ Understand and use $\sin^2\theta + \cos^2\theta = 1$	These identities may be used to solve trigonometric equations or to prove further identities.
	5.4	Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle.	Students should be able to solve equations such as $\sin(x + 70°) = 0.5$ for $0 < x < 360°$, $3 + 5\cos 2x = 1$ for $-180° < x < 180°$ $6\cos^2 x° + \sin x° - 5 = 0,\ 0 \leqslant x < 360°$ giving their answers in degrees.

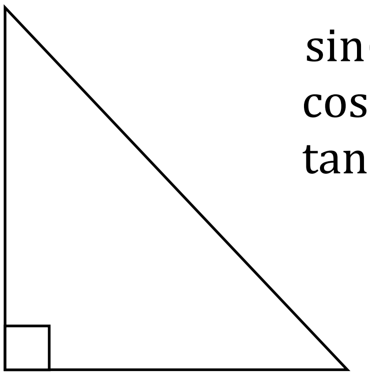
sin/cos/tan of $30°,\ 45°,\ 60°$

You will frequently encounter angles of $30°,\ 60°,\ 45°$ in geometric problems. Why?

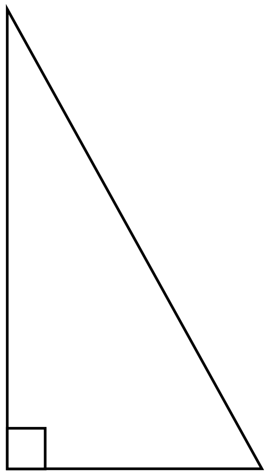
Although you will always have a calculator, you need to know how to derive these.

All you need to remember:

✏ **Draw half a unit square and half an equilateral triangle of side 2.**



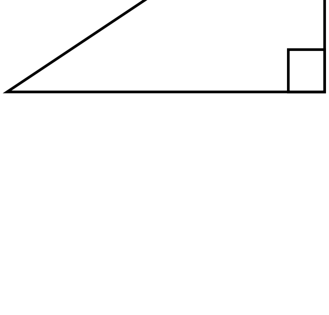
$\sin(45°) =$
$\cos(45°) =$
$\tan(45°) =$



$\sin(30°) =$
$\cos(30°) =$
$\tan(30°) =$
$\sin(60°) =$
$\cos(60°) =$
$\tan(60°) =$

The Unit Circle and Trigonometry

For values of θ in the range $0 < \theta < 90°$, you know that $\sin\theta$ and $\cos\theta$ are lengths on a right-angled triangle:



And what would be the **gradient** of the bold line (hypotenuse)?

But how do we get the rest of the graph for $sin,\ cos$ and tan when $90° \leq \theta \leq 360°$?

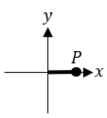
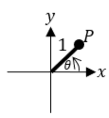
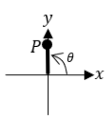
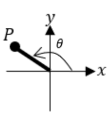
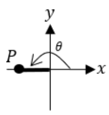
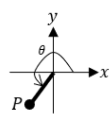
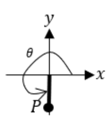
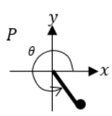
The point P on a unit circle, such that OP makes an angle θ with the positive x-axis, has coordinates $(\cos\theta, \sin\theta)$.
OP has gradient $\tan\theta$.

Angles are always measured **anticlockwise**.

We can consider the coordinate P $(\cos\theta, \sin\theta)$ as θ increases from 0 to $360°$…

Use the unit circle to determine each value in the table, **using either "0", "+ve", "-ve", "1", "-1" or "undefined"**.

x-value → $\cos\theta$; y-value → $\sin\theta$; Gradient of OP → $\tan\theta$

	$\cos\theta$	$\sin\theta$	$\tan\theta$
$\theta = 0$ 			
$0 < \theta < 90°$ 			
$\theta = 90°$ 			
$90° < \theta < 180°$ 			
$\theta = 180°$ 			
$180° < \theta < 270°$ 			
$\theta = 270°$ 			
$270° < \theta < 360°$ 			

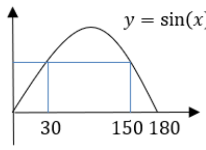
The unit circle explains the behaviour of the trigonometric graphs beyond 90°.
However, the easiest way to remember whether $\sin(x)$, $\cos(x)$, $\tan(x)$ are positive or negative is to just do a **very quick sketch (preferably mentally!)** of the corresponding graph.
Note: The textbook uses something called '*CAST diagrams*'. We will not be using them in this booklet, but you may wish to look at this technique as an alternative approach to various problems in the chapter.

<u>A Few Trigonometric Angle Laws</u>

The following are all easily derivable using a quick sketch of a trigonometric graph, and are merely a <u>convenience</u> so you don't always have to draw out a graph every time.
You are highly encouraged to **memorise these** so that you can do exam questions faster.

1 $\sin(x) = \sin(180° - x)$

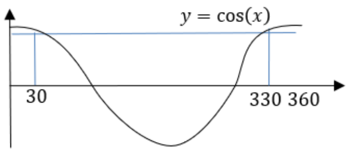
e.g. $\sin(150°) = \sin(30°)$



We saw this in the previous chapter when covering the 'ambiguous case' when using the sine rule.

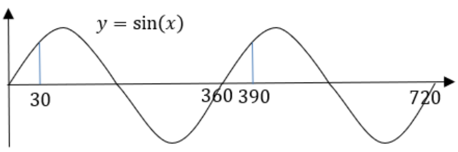
2 $\cos(x) = \cos(360° - x)$

e.g. $\cos(330°) = \cos(30°)$



3 *sin* and *cos* repeat every 360° but *tan* every 180°

e.g. $\sin(390°) = \sin(30°)$



4 $\sin(x) = \cos(90° - x)$

e.g. $\sin(50°) = \cos(40°)$

Remember from the previous chapter that "cosine" by definition is the sine of the "complementary" angle. This was/is never covered in the textbook but caught everyone by surprise when it came up in a C3 exam.

Examples

Without a calculator, work out the value of each below:

$\tan(225°) =$

$\tan(210°) =$

$\sin(150°) =$

$\cos(300°) =$

$\sin(-45°) =$

$\cos(750°) =$

$\cos(120°) =$

$\cos(315°) =$

$\sin(420°) =$

$\tan(-120°) =$

$\tan(-45°) =$

$\sin(135°) =$

Exercise 10A and B Pg 207 and 209