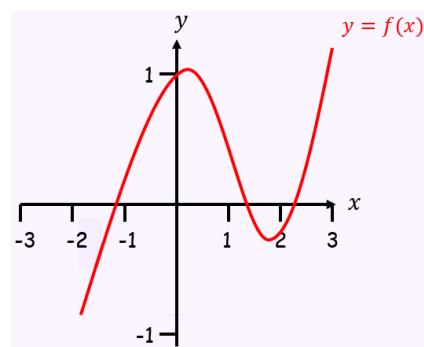


10A Roots within Intervals

1. The diagram shows a sketch of the curve $y = f(x)$, where $f(x) = x^3 - 4x^2 + 3x + 1$
 - a) Explain how the graph shows that $f(x)$ has a root between $x = 2$ and $x = 3$.



- b) Show that $f(x)$ has a root between $x = 1.4$ and $x = 1.5$

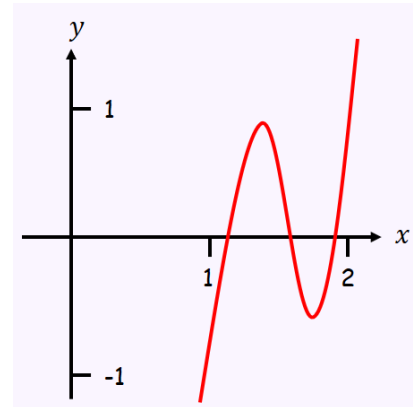
2. The graph of the function

$$f(x) = 54x^3 - 225x^2 + 309x - 140$$

is shown in the diagram.

A student observes that $f(1.1)$ and $f(1.6)$ are both negative and states that $f(x)$ has no roots in the interval $[1.1, 1.6]$

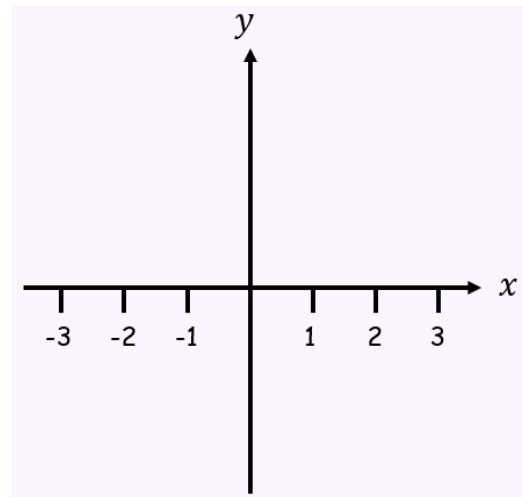
- a) Explain, referring to the diagram, why the student is incorrect



- b) Calculate $f(1.3)$, $f(1.5)$ and $f(1.7)$ and use your answer to explain why there are at least 3 roots in the interval $1.1 < x < 1.7$.

3.

- a) Using the same axes, sketch the graphs of $y = \ln x$ and $y = \frac{1}{x}$. Explain how your diagram shows that the function $f(x) = \ln x - \frac{1}{x}$ has only one root



- b) Show that this root lies in the interval $1.7 < x < 1.8$

- c) Given that the root of $f(x) = \alpha$, show that $\alpha = 1.763$ correct to 3 decimal places