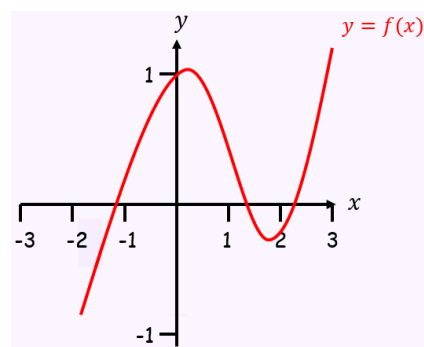


10A Roots within Intervals

1. The diagram shows a sketch of the curve $y = f(x)$, where $f(x) = x^3 - 4x^2 + 3x + 1$
 - a) Explain how the graph shows that $f(x)$ has a root between $x = 2$ and $x = 3$.



- b) Show that $f(x)$ has a root between $x = 1.4$ and $x = 1.5$

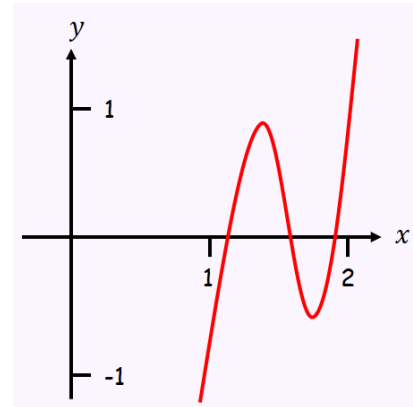
2. The graph of the function

$$f(x) = 54x^3 - 225x^2 + 309x - 140$$

is shown in the diagram.

A student observes that $f(1.1)$ and $f(1.6)$ are both negative and states that $f(x)$ has no roots in the interval $[1.1, 1.6]$

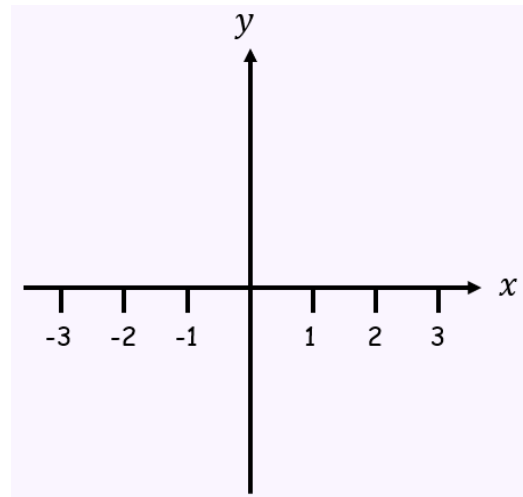
- a) Explain, referring to the diagram, why the student is incorrect



- b) Calculate $f(1.3)$, $f(1.5)$ and $f(1.7)$ and use your answer to explain why there are at least 3 roots in the interval $1.1 < x < 1.7$.

3.

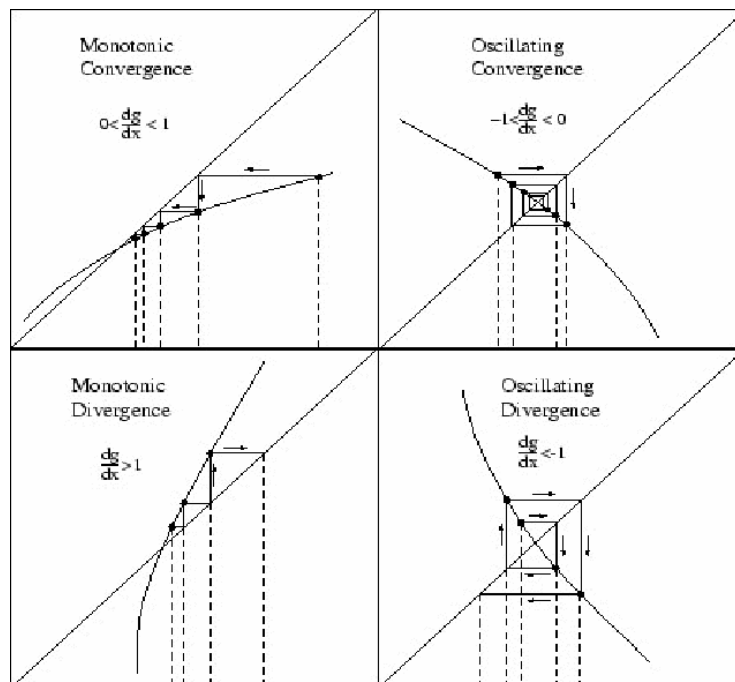
- a) Using the same axes, sketch the graphs of $y = \ln x$ and $y = \frac{1}{x}$. Explain how your diagram shows that the function $f(x) = \ln x - \frac{1}{x}$ has only one root



- b) Show that this root lies in the interval $1.7 < x < 1.8$

- c) Given that the root of $f(x) = \alpha$, show that $\alpha = 1.763$ correct to 3 decimal places

10B Iterations



1. Given that $f(x) = x^2 - 4x + 1$

a) Show that the equation $f(x) = 0$ can be written as $x = 4 - \frac{1}{x}$, $x \neq 0$.

The equation $f(x)$ has a root, α , in the interval $3 < x < 4$.

b) Use the iterative formula

$x_{n+1} = 4 - \frac{1}{x_n}$, with $x_0 = 3$ to find the value of x_1 , x_2 and x_3 .

2.

$$f(x) = x^3 - 3x^2 - 2x + 5$$

a) Show that the equation $f(x) = 0$ has a root in the interval $3 < x < 4$

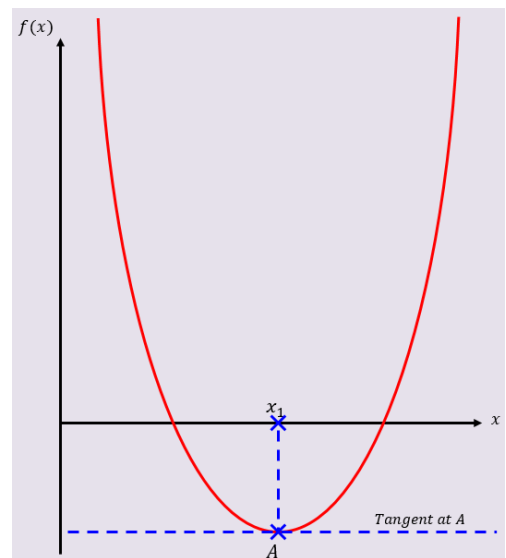
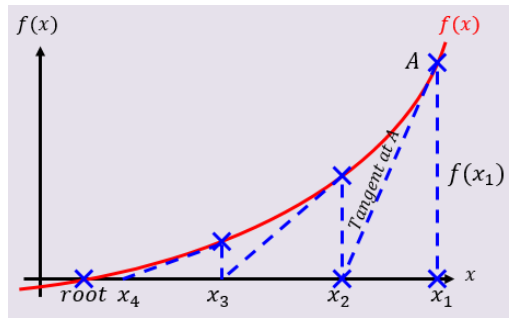
b) Use the iterative formula:

$$x_{n+1} = \sqrt{\frac{x_n^3 - 2x_n + 5}{3}}$$

To calculate the values of x_1 , x_2 and x_3 , giving answers to 4 decimal places, when:

i) $x_0 = 1.5$ ii) $x_0 = 4$

10C Newton Raphson

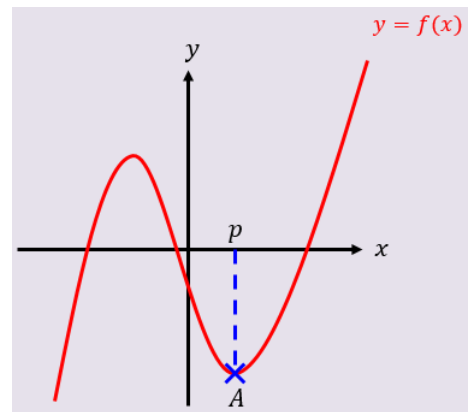


1. The diagram shows part of the curve with equation $y = f(x)$, where $f(x) = x^3 + 2x^2 - 5x - 4$.

The point A , with x -coordinate p , is a stationary point on the curve.

The equation $f(x)=0$ has a root, α , in the interval $1.8 < \alpha < 1.9$.

- a) Explain why $x_0 = p$ is not suitable to use as a first approximation to α when applying the Newton-Raphson method to $f(x)$



- b) Using $x_0 = 2$ as a first approximation to α , apply the Newton-Raphson method procedure twice to find a new approximation for α , to 3dp.

- c) By considering the change of sign in $f(x)$ over an appropriate interval, show that your answer to part b is accurate to 3 decimal places

10D Application of Numerical methods in context

1. The price of a car in £s, x years after purchase, is modelled by the function:

$$f(x) = 15000(0.85)^x - 1000\sin x, \quad x > 0$$

- a) Use the model to find the value of the car 10 years after purchase

- b) Show that $f(x)$ has a root between 19 and 20

- c) Find $f'(x)$

d) Taking 19.5 as a first approximation, apply the Newton-Raphson method once to find a second approximation for the time when the value of the car is zero. Give your answer to 3dp

e) Criticise this model with respect to the value of the car as it gets older