Upper 6 Chapter 10

Numerical Methods

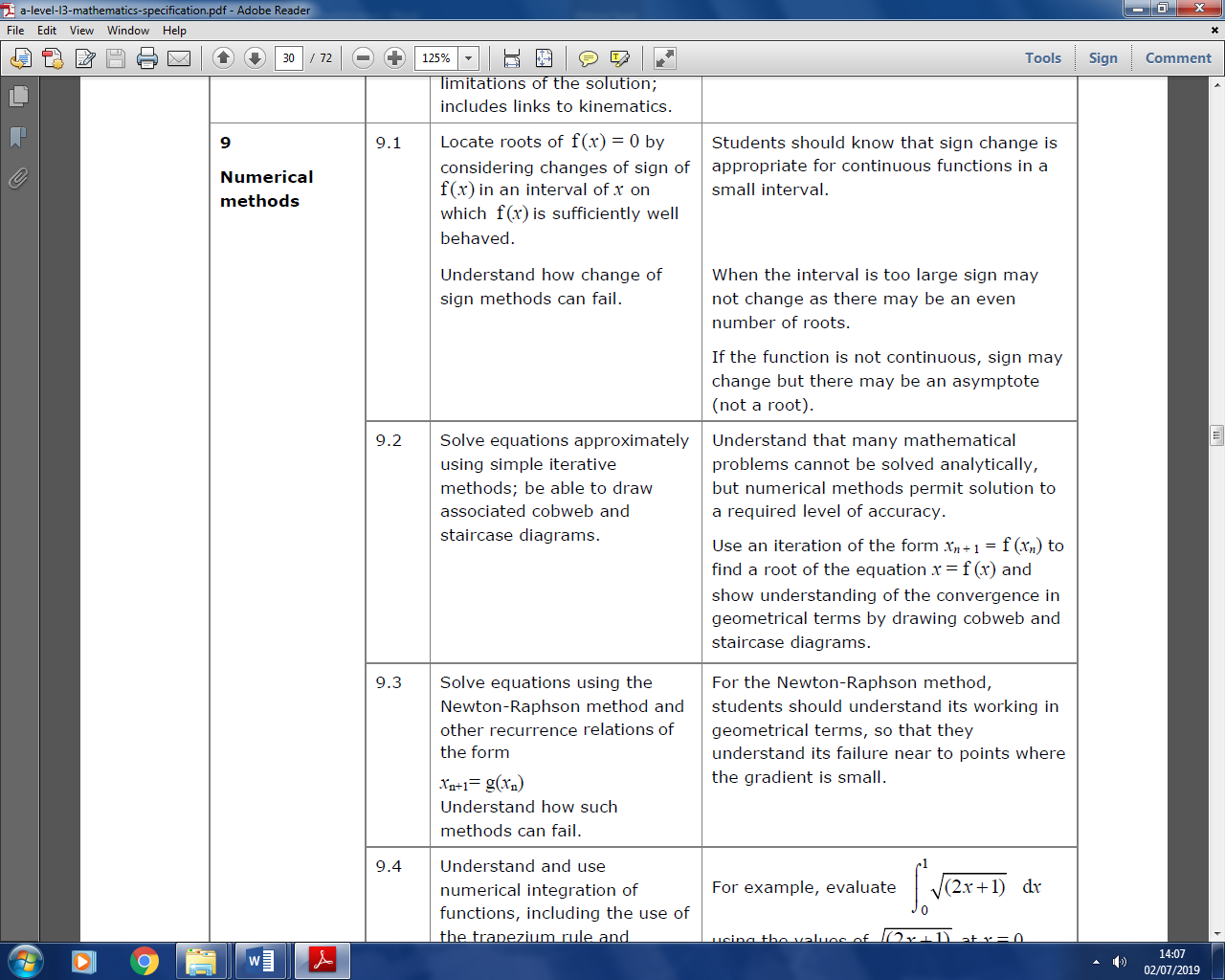
Chapter Overview

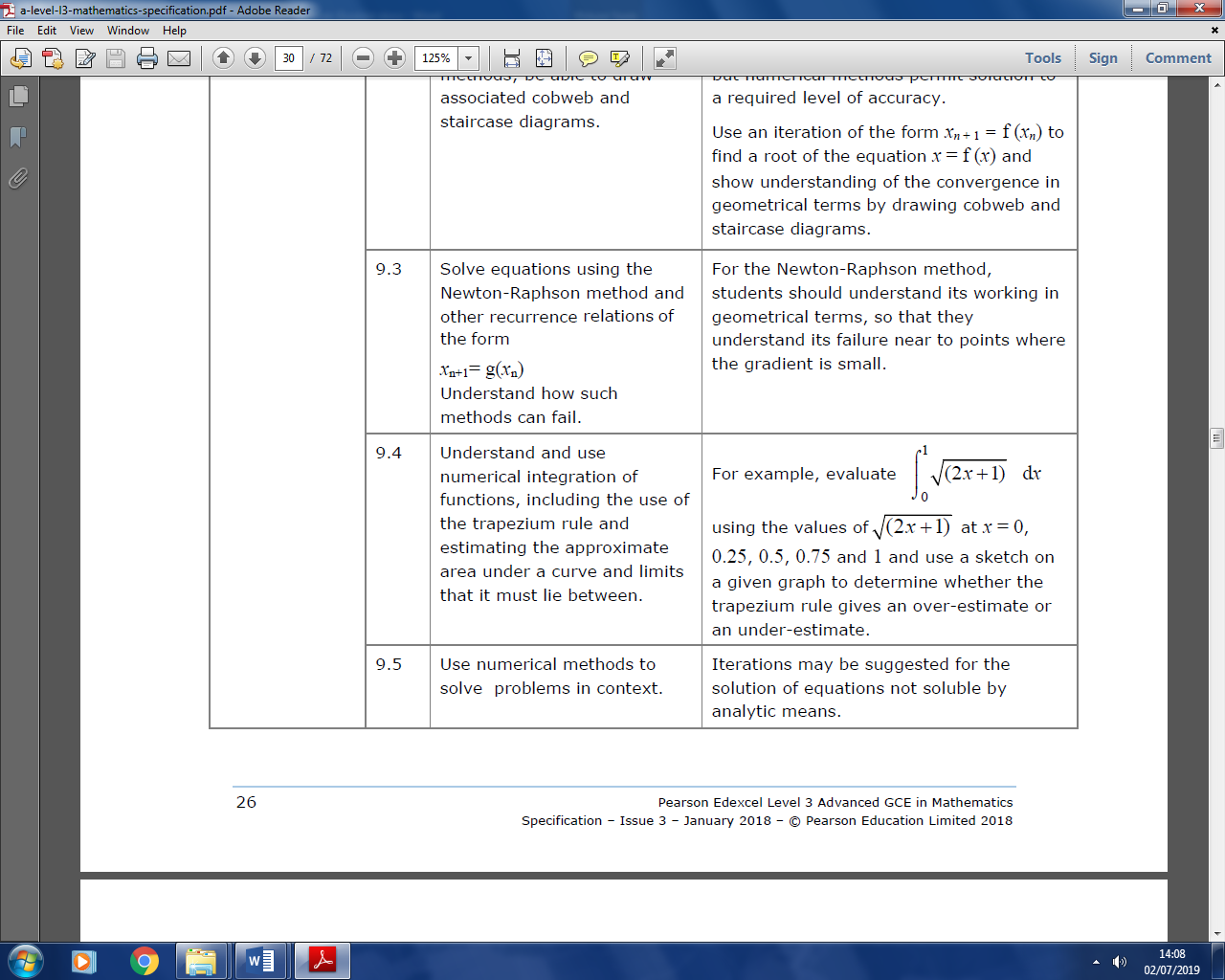
1. Locating Roots

2. Iteration

3. The Newton-Raphson Method

4. Applications to Modelling





**LOCATING ROOTS**

Finding the root of a function is to **solve the equation**

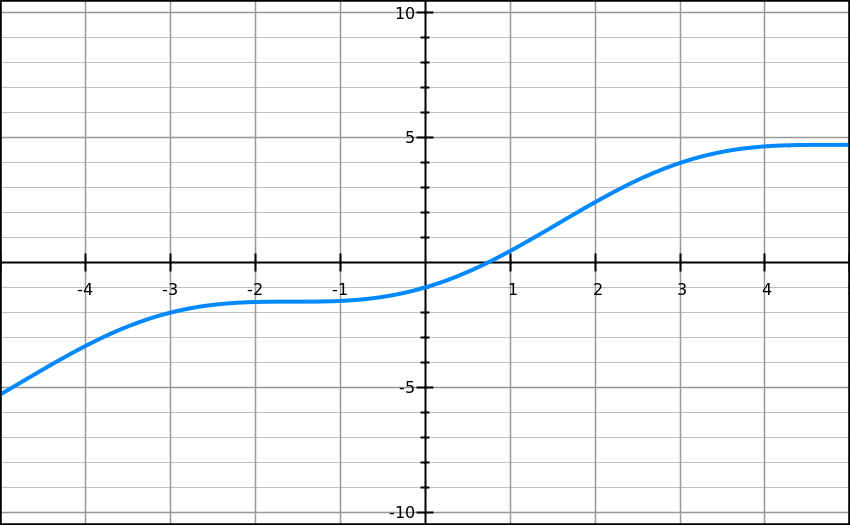
However, for some functions, the ‘exact’ root is complicated and difficult to calculate …

For example:

has the solution:

… or there is no ‘algebraic’ expression at all. (involving roots, logs, sin, cos, etc.)

For example:



has a root here

**To show that a root exists in a given interval, show that changes sign**

**Example 1**

Show that has a root between and

STEP 1: Find for the two values given in the question

STEP 2: Write a concluding statement referring to the change in sign and the fact that is a continuous function

**Note on functions that are NOT continuous:**

If the function is not continuous, the sign change may be due to an asymptote rather than a root.

For example:

When , then and .

However, although there is a sign change, a root does not exist between and

**Note on continuous functions:**

A continuous function could simply have an even number of roots in a given interval rather than no roots.

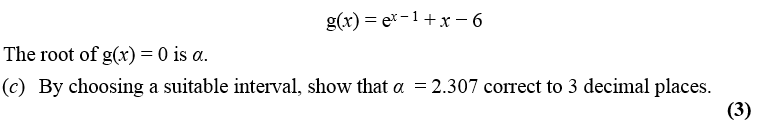
For example:

Here is negative and is also negative

However, although there are two roots, a sign change does not occur.

a

b

**Example 2**

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**Example 3**

(a) Using the same axes, sketch the graphs of and . Explain how your diagrams shows that the function has only one root.

(b) Show that this root lies in the interval

(c) Given that the root of is , show that correct to 3 decimal places.

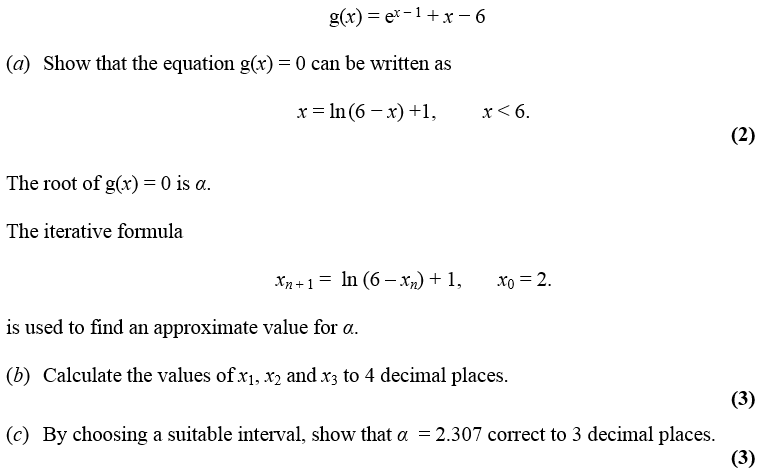
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**ITERATION**

**To solve by an iterative method, rearrange into a form and use the iterative formula**

**Example 1**

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a)

b) **, , represent successively better approximations of the root**

Initially type (i.e. 2) onto your calculator.

Now just type:

And then press your key to get successive iterations.

c)

**The starting value matters.**

* If there are a multiple roots, the iteration might converge to (i.e. approach) a different root.
* The iteration not converge to a root at all and **diverges** (i.e. approach infinity).

**Example 2**

1. Show that the equation has a root in the interval .
2. Use the iterative formula to calculate the values of , and , giving your answers to 4 decimal places, and taking:  
   (i) (ii)

**Staircase and cobweb diagrams**

**Example 3**

1. Show that the root of the equation can be written as
2. Using the iterative formula , and starting with , draw a staircase diagram, indicating on your -axis, as well as the root .

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**THE NEWTON-RAPHSON METHOD**

**The Newton- Raphson method can be used to find numerical solutions to equations of the form . You need to be able to differentiate in order to use this method.**

**The Newton- Raphson formula is:**

**Example 1**

Recall that in lesson 1 we saw that the function has aroot, , in the interval .

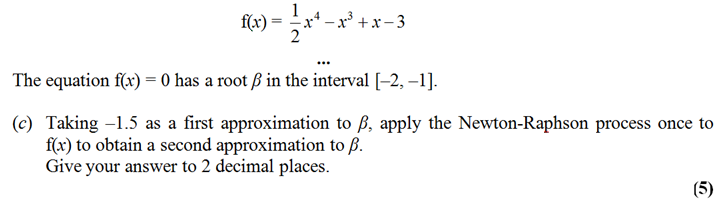
Using as a first approximation to, apply the Newton-Raphson procedure three times to find a better approximation to which, in this case, will be accurate to 7 decimal places.

To perform iterations quickly, do the following on your calculator:

[0.5] [=]

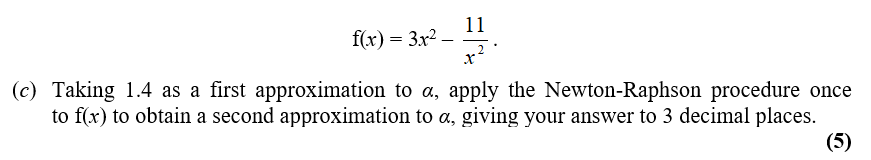
[ANS] – (ANS – cos(ANS))/(1 + sin(ANS))

Then press [=].

**Example 2**

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**Example 3**

**When does Newton-Raphson fail?**

tangent

**If the starting value was the stationary point**, then , resulting in a division by 0 in the above formula.

Graphically, it is because the tangent will never reach the -axis.

Newton-Raphson also suffers from the same drawbacks as solving by iteration, in that it is possible for the values of to **diverge**.

In this example, the oscillate either side of 0, but get gradually further away from .

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**APPLICATIONS TO MODELLING**

**Example 4**

The price of a car in £s, years after purchase, is modelled by the function

1. Use the model to find the value, to the nearest hundred £s, of the car 10 years after purchase.
2. Show that has a root between 19 and 20.
3. Find
4. Taking 19.5 as a first approximation, apply the Newton-Raphson method once to to obtain a second approximation for the time when the value of the car is zero. Give your answer to 3 decimal places.
5. Criticise this model with respect to the value of the car as it gets older.

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