Chapter 5 - Statistics

Probability Year 1

Chapter Overview

1. Basic probability
2. Venn diagrams
3. Mutually Exclusive / Independent Events
4. Tree diagrams

3 **Probability**	3.1	**Understand and use mutually exclusive and independent events when calculating probabilities.** **Link to discrete and continuous distributions.**	**Venn diagrams or tree diagrams may be used.** Set notation to describe events may be used. Use of $P(B\|A) = P(B)$, $P(A\|B) = P(A)$, $P(A \cap B) = P(A)\,P(B)$ in connection with independent events. **No formal knowledge of probability density functions is required but students should understand that area under the curve represents probability in the case of a continuous distribution.**

(Set notation **may** be used in Year 1 – will be seen more commonly in Year 2)

Probability concepts

An **experiment** is a repeatable process that gives rise a number a number of **outcomes**.

An **event** is a set of one or more of these outcomes.

(We often use capital letters to represent them)

$E =$ "rolling an even number"
$P =$ "rolling a prime number"

A **sample space** is the set of all possible outcomes.

Because we are dealing with sets, we can use a **Venn diagram**, where

- the numbers are the individual outcomes,
- the sample space is a rectangle and
- the events are sets, each a subset of the sample space.

You do not need to use set notation like ∩ and ∪ in this module (but ordinarily you would!)

Example

Two fair spinners each have four sectors numbered 1 to 4. The two spinners are spun together and the sum of the numbers indicated on each spinner is recorded.
Find the probability of the spinners indicating a sum of
(a) exactly 5 (b) more than 5

Another Example

The table shows the times taken, in minutes, for a group of students to complete a number puzzle.

Time, t (min)	$5 \leq t < 7$	$7 \leq t < 9$	$9 \leq t < 11$	$11 \leq t < 13$	$13 \leq t < 15$
Frequency	6	13	12	5	4

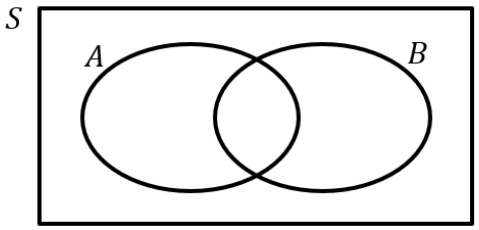
A student is chosen at random. Find the probability for a group of students to complete a number puzzle

(a) In under 9 minutes (b) in over 10.5 minutes.

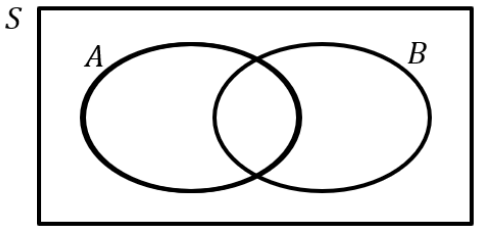
Textbook Exercise 5A Pg 71

Venn Diagrams

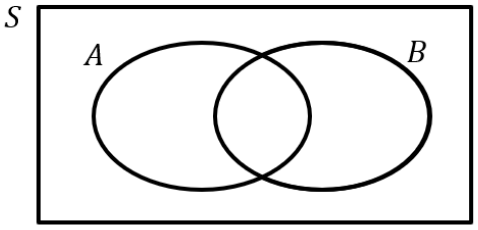
Venn Diagrams allow us to combine events, e.g. "A happened **and** B happened".



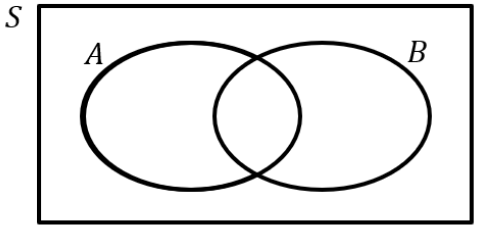
The event "A **and** B"
Known as the **intersection** of A and B.



The event "A **or** B"
Known as the **union** of A and B.



The event "not A"
Known as the **union** of A and B.



These can be combined,
e.g. "A and not B".

Example involving probabilities

We can either put frequencies or probabilities into the Venn Diagram.

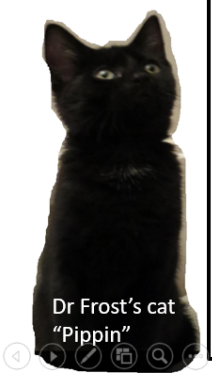
Given that $P(A) = 0.6$ and $P(A \text{ or } B) = 0.85$, find the probability of:
a) $P(\text{not } A \text{ and } B)$
b) $P(\text{neither } A \text{ nor } B)$

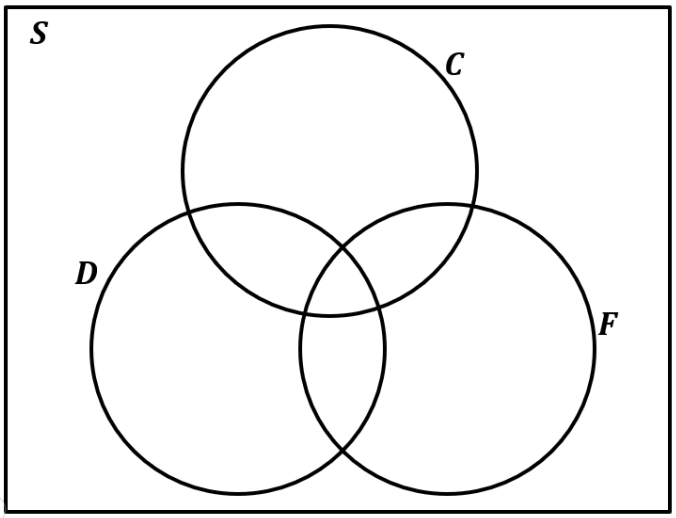
Example involving frequencies

A vet surveys 100 of her clients. She finds that
25 own dogs, 15 own dogs and cats, 11 own dogs and tropical fish, 53 own cats, 10 own cats and tropical fish, 7 own dogs, cats and tropical fish, 40 own tropical fish.

Fill in this Venn Diagram, and hence answer the following questions:

a) $P(owns\ dog\ only)$
b) $P(does\ not\ own\ tropical\ fish)$
c) $P(does\ not\ own\ dogs,\ cats, or\ tropical\ fish)$





Fro Tip: Start from the centre frequency and work your way outwards using subtraction.

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The following shows the results of a survey on the types of exercise taken by a group of 100 people.

65 run	48 swim
60 cycle	40 run and swim
30 swim and cycle	35 run and cycle
25 do all three	

(*a*) Draw a Venn Diagram to represent these data. **(4)**

Find the probability that a randomly selected person from the survey

(*b*) takes none of these types of exercise, **(2)**
(*c*) swims but does not run, **(2)**
(d) takes at least two of these types of exercise. **(2)**

Textbook Exercise 5B Pg 74

Mutually Exclusive Events

Independent Events

- If two events are independent
- If A and B are independent then:

Fro Note: Independence does not affect how the circles interact in a Venn Diagram.

Example

1 2 3 4

1 I pick one of the four numbers 1, 2, 3, 4 at random. What's the probability that:

a) I pick a multiple of 2:

b) I pick a multiple of 4:

2 Explain (conceptually) why these two events are not independent.

3 Show that the events are not independent.

$$P(multiple\ of\ 2) \times P(multiple\ of\ 4) =$$

$$P(multiple\ of\ 2\ \boldsymbol{and}\ multiple\ of\ 4) =$$

This is a common exam question. Either show that $P(A\ and\ B) = P(A) \times P(B)$ or that $P(A\ and\ B) \neq P(A) \times P(B)$

Further Examples

[Textbook] Events A and B are mutually exclusive and $P(A) = 0.2$ and $P(B) = 0.4$.

a) Find $P(A \text{ or } B)$

b) Find $P(A \text{ but not } B)$

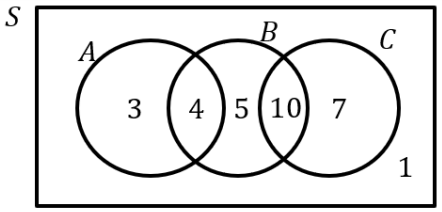
c) Find $P(\text{neither } A \text{ nor } B)$

[Textbook] Events A and B are independent and $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$. Find $P(A \text{ and } B)$.

[Textbook] The Venn diagram shows the number of students in a particular class who watch any of three popular TV programmes.

a) Find the probability that a student chosen at random watches B or C or both.

b) Determine whether watching A and watching B are statistically independent.



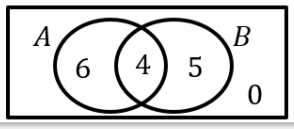
Test Your Understanding

There are three events A, B, C. The events A and B are mutually exclusive.

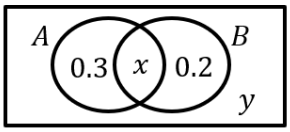
a) Draw a Venn diagram which represents this information.

b) If $P(A) = 0.1$ and $P(B) = 0.6$, determine $P(neither\ A\ nor\ B)$

The Venn diagram shows the number of people who like each of two different colours. Determine if A and B are independent.



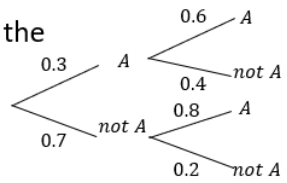
The Venn diagram shows the probability of each event. Given that A and B are independent, determine the possible values of x.



Textbook Exercise 5C pg 77

Tree Diagrams

At GCSE we saw that tree diagrams were an effective way of showing the outcome of two events which happen **in succession**.
(Personal opinion however is that their use is easily avoidable)



There are 3 yellow and 2 green counters in a bag. I take two counters at random. Determine the probability that:

a) They are of the same colour.
b) They are of different colours.

The probability I hit a target on each shot is 0.3. I keep firing until I hit the target. Determine the probability I hit the target on the 5^{th} shot.

Textbook Exercise 5D Pg 79