The Shortest Distance Between **Parallel** Lines

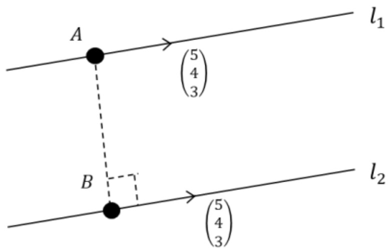
The key with such "shortest distance" problems is that the line connecting l_1 and l_2 whose distance is shortest, is perpendicular to the two lines.

Example

Show that the shortest distance between the parallel lines with equations:

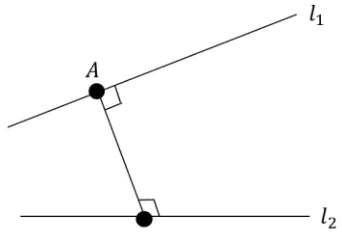
$\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k} + \lambda(5\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k})$ and $\boldsymbol{r} = 2\boldsymbol{i} + \boldsymbol{k} + \mu(5\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k})$,

Where λ and μ are scalars, is $\frac{21\sqrt{2}}{10}$



The Shortest Distance Between **Any** Lines

Again, use same strategy, but this time $\overrightarrow{AB}$ is perpendicular to **both** l_1 and l_2.



Example

The lines l_1 and l_2 have equations $r = \begin{pmatrix}1\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\1\\1\end{pmatrix}$ and $r = \begin{pmatrix}-1\\3\\-1\end{pmatrix} + \mu\begin{pmatrix}2\\-1\\-1\end{pmatrix}$ respectively, where λ and μ are scalars.

Find the shortest distance between these two lines.

<u>The Shortest Distance Between a Point and a Line</u>

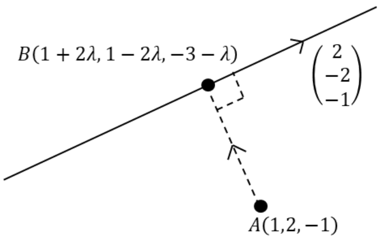
Again, same strategy! If B is a point on the line, $\overrightarrow{AB}$ is perpendicular to the direction of the line.

Example

The line l has equation $\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z+3}{-1}$, and the point A has coordinates $(1,2,-1)$.

(a) Find the shortest distance between A and l.

Find the Cartesian equation of the line that is perpendicular to l and passes through A



Ex 9F Q 1-5, 7, 9, 11