The Shortest Distance Between **Parallel** Lines

The key with such “shortest distance” problems is that the line connecting $l\_{1}$ and $l\_{2}$ whose distance is shortest, is perpendicular to the two lines.

Example

Show that the shortest distance between the parallel lines with equations:

$r=i+2j-k+λ\left(5i+4j+3k\right)$ and $r=2i+k+μ\left(5i+4j+3k\right)$,

Where $λ$ and $μ$ are scalars, is $\frac{21\sqrt{2}}{10}$

The Shortest Distance Between **Any** Lines

Again, use same strategy, but this time $\vec{AB}$ is

perpendicular to **both** $l\_{1}$ and $l\_{2}$.

Example

The lines $l\_{1}$ and $l\_{2}$ have equations $r=\left(\begin{matrix}1\\0\\0\end{matrix}\right)+λ\left(\begin{matrix}0\\1\\1\end{matrix}\right)$ and $r=\left(\begin{matrix}-1\\3\\-1\end{matrix}\right)+μ\left(\begin{matrix}2\\-1\\-1\end{matrix}\right)$ respectively, where $λ$ and $μ$ are scalars.

Find the shortest distance between these two lines.

The Shortest Distance Between a Point and a Line

Again, same strategy! If $B$ is a point on the line, $\vec{AB}$ is perpendicular to the direction of the line.

Example

The line $l$ has equation $\frac{x-1}{2}=\frac{y-1}{-2}=\frac{z+3}{-1}$, and the point $A$ has coordinates $\left(1,2,-1\right)$.

1. Find the shortest distance between $A$ and $l$.

Find the Cartesian equation of the line that is perpendicular to $l$ and passes through $A$



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