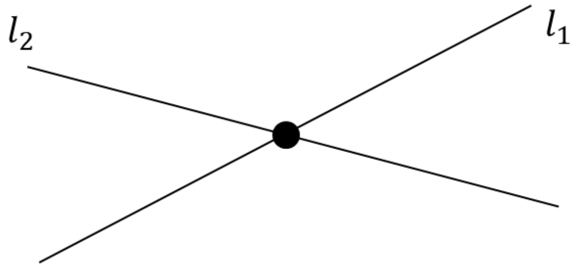
Points of Intersection

Example

The lines l_1 and l_2 have vector equations $r = 3i + j + k + \lambda(i - 2j - k)$ and $r = -2j + 3k + \mu(-5i + j + 4k)$ respectively. Show that the two lines intersect, and find the position vector of the point of intersection.



We can represent any point on l_1 as the position vector $\begin{pmatrix} 3+\lambda \\ 1-2\lambda \\ 1-\lambda \end{pmatrix}$ and any point line l_2 as $\begin{pmatrix} -5\mu \\ -2+\mu \\ 3+4\mu \end{pmatrix}$. If the lines intersect, there must be a choice of λ and μ that makes those two points equal,

i.e. $\begin{pmatrix} 3+\lambda \\ 1-2\lambda \\ 1-\lambda \end{pmatrix} = \begin{pmatrix} -5\mu \\ -2+\mu \\ 3+4\mu \end{pmatrix}$

Test Your Understanding

[June 2007 Q5] 5. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(a) Show that l_1 and l_2 do not meet. **(4)**

The Intersection of a Line and a Plane

Find the point of intersection of the line l and the plane Π where:

$$l: \ \boldsymbol{r} = -\boldsymbol{i} + \boldsymbol{j} - 5\boldsymbol{k} + \lambda(\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}) \Pi: \boldsymbol{r} \cdot (\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) = 4$$

The Intersection in Cartesian Form

The lines l_1 and l_2 have equations $\frac{x-2}{4} = \frac{y+3}{2} = z - 1$ and $\frac{x+1}{5} = \frac{y}{4} = \frac{z-4}{-2}$ respectively. Prove that l_1 and l_2 are skew.

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