Scalar Product

The scalar/dot product $\boldsymbol{a} \cdot \boldsymbol{b}$ of two vectors is the sum of the products of the components. The result is a **scalar**, hence the name.

$$\boldsymbol{a} \cdot \boldsymbol{b} = \sum a_i b_i$$

Examples

1. $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} =$

2. $\begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} =$

3. $\begin{pmatrix} a \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ b \\ 10 \end{pmatrix} =$

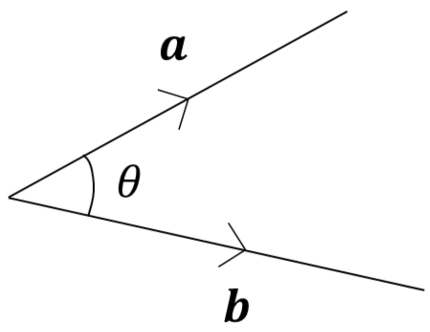
Remarkably, if the two vectors are unit vectors, the dot product gives us the cosine of the angle between them.

Angle between vectors:

$$\frac{\boldsymbol{a}}{|\boldsymbol{a}|} \cdot \frac{\boldsymbol{b}}{|\boldsymbol{b}|} = \cos\theta$$

or

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$



Note the direction if the vectors and the corresponding angle.

Example

Find the acute angle between the vectors $\boldsymbol{a} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix}$.

Example

Find the angle between the vectors $\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix}$.

Example

If $A(2,3,5)$, $B(5,0,4)$ and $C(4,-3,2)$, determine the angle ABC.

Hence find the area of triangle ABC.

Perpendicular Vectors

Given that

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

It follows that if two vectors are perpendicular then ……………………………………………

We can use this to prove that two vectors are perpendicular.

Example

1. Show that $\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ are perpendicular.

2. Given that $\boldsymbol{a} = -2\boldsymbol{i} + 5\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{b} = 4\boldsymbol{i} - 8\boldsymbol{j} + 5\boldsymbol{k}$, find a vector which is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$.

<u>Test Your Understanding</u>

[June 2008 Q6] 6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$
$$l_2 : \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(*b*) Show that l_1 and l_2 are perpendicular to each other. **(2)**

[Jan 2012 Q7] 7. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$, the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$, and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(*a*) Find the vector $\overrightarrow{AB}$. **(2)**

(*b*) Find a vector equation for the line l. **(2)**

(*c*) Show that the size of the angle BAD is 109°, to the nearest degree. **(4)**

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(*d*) Find the position vector of C. **(2)**

(*e*) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures. **(3)**

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