

Cartesian Form of Equation of a Straight Line



Examples

1. Find the Cartesian equation of the line with equation $\mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$.

2. Find the Cartesian equation of the line with equation $\mathbf{r} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$.

3. The Cartesian equation of a line is $y = 3x + 2$. Find the vector form of the equation of the line.

4. The Cartesian equation of a line is $\frac{x-2}{3} = \frac{y+5}{1} = \frac{z}{4}$. Find the vector form of the equation of the line.

The Equation of a Plane

The equation of a plane can be written in vector form.

Let the point R, with position vector \mathbf{r} be an arbitrary point on the plane Π .

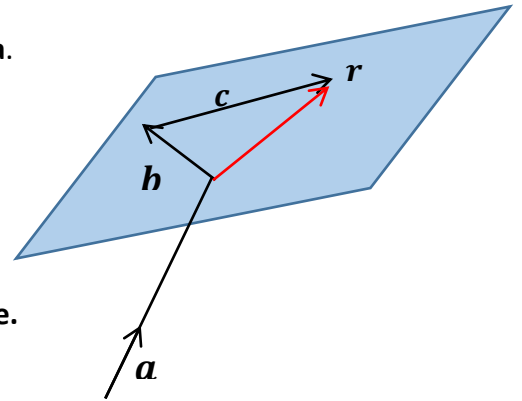
Suppose Π passes through the point A, with position vector \mathbf{a} .

Let \mathbf{b} and \mathbf{c} be non-parallel vectors in the plane.

The position of the general point R can be found by

$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

This is one of several ways of writing the equation of a plane.



Example

A plane Π passes through the points $A(2,6,-1)$, $B(7,2,-1)$, $C(4,2,5)$

Find the equation of the plane Π in the form $\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

Example

Verify that the point P with position vector $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ lies in the plane with vector equation

$$r = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Test Your Understanding

[June 2015 Q5] The points A , B and C have position vectors $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ respectively.

The plane Π contains the points A , B and C .

(c) Find a vector equation of Π

(4)