Core Pure 1

Vectors

Chapter Overview

1: Equations of straight lines in 3D

2: Equations of planes

3: Scalar product and angles between line + line or plane + line or plane + plane.

4: Scalar product form of equation of plane

5: Point of intersection of two planes

6: Perpendicular distance between line + line <u>or</u> point + line <u>or</u> point + plane

6 **Further vectors**	6.1	**Understand and use the vector and Cartesian forms of an equation of a straight line in 3-D.**	**The forms, r = a + λb and** $\frac{x-a_1}{b_1} = \frac{x-a_2}{b_2} = \frac{x-a_3}{b_3}$ **Find the point of intersection of two straight lines given in vector form.** **Students should be familiar with the concept of skew lines and parallel lines.**
	6.2	**Understand and use the vector and Cartesian forms of the equation of a plane.**	**The forms** **r = a + λb + μc and ax + by + cz = d**
	6.3	**Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane.**	$\text{a.b} = \lvert\text{a}\rvert\,\lvert\text{b}\rvert\cos\theta$ **The form r.n = k for a plane.**
	6.4	**Check whether vectors are perpendicular by using the scalar product.**	**Knowledge of the property that a.b = 0 if the vectors a and b are perpendicular.**

6 **Further vectors** *continued*	6.5	**Find the intersection of a line and a plane.** **Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane.**	**The perpendicular distance from** (α, β, γ) **to** $n_1x + n_2y + n_3z + d = 0$ **is** $\frac{\lvert n_1\alpha + n_2\beta + n_3\gamma + d\rvert}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

The Vector Equation of a Line

Important understanding points:

- $\boldsymbol{a}$ and $\boldsymbol{b}$ are constants (i.e. fixed for a given line) while λ is a variable.
- It is often helpful to write as a single vector, e.g:

$$\begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 3+\lambda \\ \lambda \\ -2 \end{pmatrix}$$

- It is very important the you can distinguish between the **position vector** $\boldsymbol{r}$ of a **point on the line**, and the **direction** $\boldsymbol{b}$ of the line

Examples

1. The equation of line l_1 is $\boldsymbol{r} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

   Find the vector equation of a line parallel to l_1 which passes through the point $(2,5,1)$.

2. The equation of line l_1 is $\boldsymbol{r} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

   Find the coordinates of the points on l_1 which are a distance of 3 away from $(3,4,4)$.

3. Find a vector equation of the straight line which passes through the point A, with position vector $5\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$ and is parallel to the vector $2\boldsymbol{i} - 3\boldsymbol{k}$.

4. Find a vector equation of the straight line which passes through the points A and B, with coordinates $(4,5,-1)$ and $(6,3,2)$ respectively.

5. The straight line has vector equation $r = (3i + 2j - 5k) + t(i - 6j - 2k)$. Given that the point $(a, b, 0)$ lines on l, find the value of a and the value of b.

6. The straight line l has vector equation $\boldsymbol{r} = (2i + 5j - 3k) + \lambda(6i - 2j + 4k)$.
Show that another vector equation of l is

$$\boldsymbol{r} = (8i + 3j + k) + \mu(3i - j + 2k)$$

Test Your Understanding

Find a vector equation of the straight line which passes through the points A and B, with coordinates $(3,0,2)$ and $(-6,1,0)$ respectively.

Final Example

The line l has equation $r = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, and the point P has position vector $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

(a) Show that P does not lie on l.

Given that a circle, centre P, intersects l at points A and B, and that A has position vector $\begin{pmatrix} 0 \\ -3 \\ 6 \end{pmatrix}$,

(b) find the position vector of B.