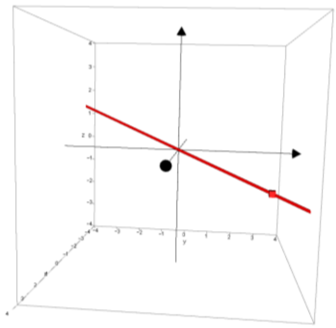
9A Part 1 3D Lines Introduction



1. Find the equation of the straight line that passes through the point A, which has position vector $\begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}$, and is parallel to the vector $\begin{pmatrix} 7 \\ 0 \\ -3 \end{pmatrix}$.

2. Find a vector equation of the straight line that passes through the points A and B, with coordinates $(4,5,-1)$ and $(6,3,2)$ respectively.

3. The straight line l has vector equation:

$$\boldsymbol{r} = (3\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k}) + t(\boldsymbol{i} - 6\boldsymbol{j} - 2\boldsymbol{k})$$

Given that the point $(a, b, 0)$ lies on l, find the value of a and the value of b.

4. The straight line l has vector equation:

$$\boldsymbol{r} = (2\boldsymbol{i} + 5\boldsymbol{j} - \boldsymbol{3k}) + \lambda(6\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k})$$

Show that another vector equation of l is:

$$\boldsymbol{r} = (8\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}) + \mu(3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k})$$

9A Part 2 Cartesian 3D Lines

1. With respect to the fixed origin O, the line l is given by the equation:

$$\boldsymbol{r} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

   Prove that a Cartesian form of the equation of l is:

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3}$$

2. Find a Cartesian equation of the line with equation:

$$\boldsymbol{r} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$$

3. The line l has equation:

$$\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

The point P has position vector:

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

a) Show that P does not line on l

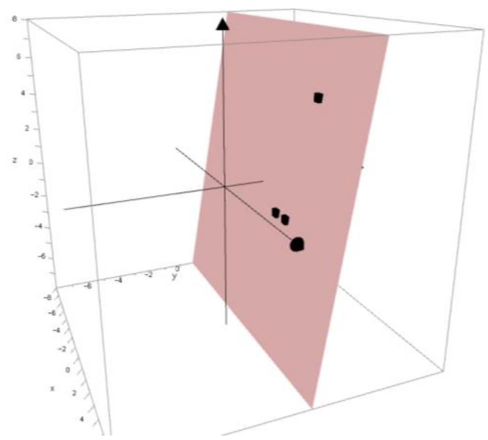
b) Given that a circle, centre P, intersects l at points A and B, and that A has position vector:

$$A = \begin{pmatrix} 0 \\ -3 \\ 6 \end{pmatrix}$$

Find the position vector of B.

<u>9B Part 1 3D Planes Introduction</u>

1. Find, in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$, an equation of the plane that passes through the points $A(2,2,-1)$, $B(3,2,-1)$ and $C(4,3,5)$



2. Verify that the point P with position vector $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ lies in the plane with vector equation:

$$\boldsymbol{r} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

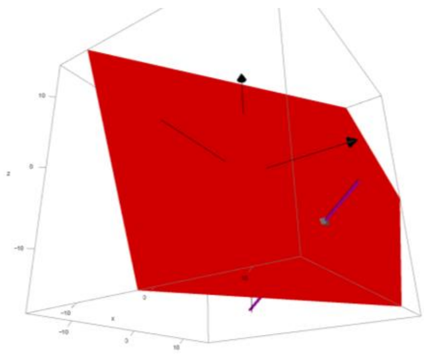
9B Part 2 Cartesian 3D Planes

2D notes:

1. The straight line graph has normal vector $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ and passes through $(2,3)$. Find the equation of the line.

3D notes:

2. The plane Π is perpendicular to the normal vector $\boldsymbol{n} = 3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ and passes through the point P with position vector $8\boldsymbol{i} + 4\boldsymbol{j} - 7\boldsymbol{k}$. Find a Cartesian equation of Π.



9C Scalar Products & Angles Between Lines

1. Given that $\boldsymbol{a} = \begin{pmatrix} 8 \\ -5 \\ -4 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix}$.

a) Find $\boldsymbol{a}.\boldsymbol{b}$

b) Find the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$, giving your answer in degrees to 1 decimal place

2. Given that the vectors $\boldsymbol{a} = 2\boldsymbol{i} - 6\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = 5\boldsymbol{i} + 2\boldsymbol{j} + \lambda\boldsymbol{k}$ are perpendicular, find the value of λ.

3. Given that $\boldsymbol{a} = -2\boldsymbol{i} + 5\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{b} = 4\boldsymbol{i} - 8\boldsymbol{j} + 5\boldsymbol{k}$, find a vector which is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$.

4. The points A, B and C have coordinates $(2,-1,1)$, $(5,1,7)$ and $(6,-3,1)$ respectively.

a) Find $\overrightarrow{AB}.\overrightarrow{AC}$

b) Hence, or otherwise, find the area of triangle ABC

9D Acute Angles Between Lines & Planes

1. The lines l_1 and l_2 have vector equations:

$$\boldsymbol{r} = (2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) + t(3\boldsymbol{i} - 8\boldsymbol{j} - \boldsymbol{k})$$

and

$$\boldsymbol{r} = (7\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}) + s(2\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$$

Given that l_1 and l_2 intersect, find the size of the acute angle between the lines, to 1 decimal place.

r.n = k for equation of a plane notes

2. The plane Π passes through the point A and is perpendicular to the vector $\boldsymbol{n}$.

   Given that $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}$ and $\boldsymbol{n} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$, with O being the origin, find an equation of the plane:

a) In scalar product form

b) In Cartesian form

3. Find the acute angle between the line $\boldsymbol{l}$ with equation:

$$\boldsymbol{r} = 2\boldsymbol{i} + \boldsymbol{j} - 5\boldsymbol{k} + \lambda(3\boldsymbol{i} + 4\boldsymbol{j} - 12\boldsymbol{k})$$

and the plane with equation:

$$\boldsymbol{r}.(2\boldsymbol{i} - 2\boldsymbol{j} - \boldsymbol{k}) = 2$$

4. Find the acute angle between the planes with equations $\boldsymbol{r}.\begin{pmatrix} 4 \\ 4 \\ -7 \end{pmatrix} = 13$ and $\boldsymbol{r}.\begin{pmatrix} 7 \\ -4 \\ 4 \end{pmatrix} = 6$.

9E Points of Intersection

1. The lines l_1 and l_2 have vector equations:

$$\boldsymbol{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$$

and

$$\boldsymbol{r} = \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 1 \\ 4 \end{pmatrix}$$

Show that the lines intersect, and find their point of intersection.

2. Find the coordinates of the point of intersection of the line l and the plane Π where l has equation:

$$\boldsymbol{r} = -\boldsymbol{i} + \boldsymbol{j} - 5\boldsymbol{k} + \lambda(\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k})$$

And Π has equation:

$$\boldsymbol{r}.(\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) = 4$$

3. The lines l_1 and l_2 have equations:

$$\frac{x-2}{4} = \frac{y+3}{2} = z - 1$$

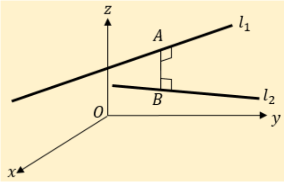
and

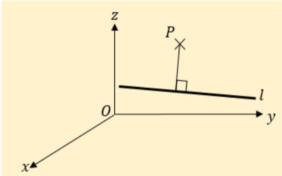
$$\frac{x+1}{5} = \frac{y}{4} = \frac{z-4}{-2}$$

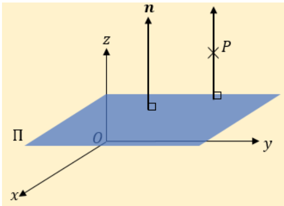
respectively.

Prove that l_1 and l_2 are skew.

9F Part 1 Perpendicular Distances with Lines







1. Show that the shortest distance between the parallel lines with equations:

$$r = i + 2j - k + \lambda(5i + 4j + 3k)$$

and

$$r = 2i + k + \mu(5i + 4j + 3k)$$

is $\frac{21\sqrt{2}}{10}$

2. The lines l_1 and l_2 have equations:

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \qquad \boldsymbol{r} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

Find the shortest distance between these two lines.

3. The line l has equation:

$$\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z+3}{-1}$$

The point A has coordinates $(1,2,-1)$

a) Find the shortest distance between A and l.

b) Find a Cartesian equation of the line that is perpendicular to l, and passes through A.

9F Part 2 Perpendicular Distances with Lines & Planes

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\dfrac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$.

1. Find the perpendicular distance from the point with coordinates $(3,2,-1)$ to the plane with equation $2x - 3y + z = 5$

2. The plane Π has equation:

$$\boldsymbol{r}.(\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}) = 5$$

The point P has coordinates:

$$(1,3,-2)$$

a) Find the shortest distance between P and Π

b) The point Q is a reflection of P in Π. Find the coordinates of Q.

3. The line l_1 has equation:

$$\frac{x-2}{2} = \frac{y-4}{-2} = \frac{z+6}{1}$$

The plane Π has equation:

$$2x - 3y + z = 8$$

The line l_2 is a reflection of l_1 in the plane Π. Find a vector equation of the line l_2.