**9A Part 1 3D Lines Introduction**



1. Find the equation of the straight line that passes through the point A, which has position vector $\left(\begin{matrix}3\\-5\\4\end{matrix}\right)$, and is parallel to the vector $\left(\begin{matrix}7\\0\\-3\end{matrix}\right)$.
2. Find a vector equation of the straight line that passes through the points A and B, with coordinates $\left(4,5,-1\right)$ and $\left(6,3,2\right)$ respectively.
3. The straight line $l$ has vector equation:

$$r=\left(3i+2j-5k\right)+t(i-6j-2k)$$

Given that the point $(a,b,0)$ lies on $l$, find the value of $a$ and the value of $b$.

1. The straight line $l$ has vector equation:

$$r=\left(2i+5j-3k\right)+λ(6i-2j+4k)$$

Show that another vector equation of $l$ is:

$$r=\left(8i+3j+k\right)+μ(3i-j+2k)$$

**9A Part 2 Cartesian 3D Lines**

1. With respect to the fixed origin O, the line $l$ is given by the equation:

$$r=\left(\begin{matrix}a\_{1}\\a\_{2}\\a\_{3}\end{matrix}\right)+λ\left(\begin{matrix}b\_{1}\\b\_{2}\\b\_{3}\end{matrix}\right)$$

Prove that a Cartesian form of the equation of $l$ is:

$$\frac{x-a\_{1}}{b\_{1}}=\frac{y-a\_{2}}{b\_{2}}=\frac{z-a\_{3}}{b\_{3}}$$

1. Find a Cartesian equation of the line with equation:

$$r=\left(\begin{matrix}4\\3\\-2\end{matrix}\right)+λ\left(\begin{matrix}-1\\2\\5\end{matrix}\right)$$

1. The line $l$ has equation:

$$r=\left(\begin{matrix}-2\\1\\4\end{matrix}\right)+λ\left(\begin{matrix}1\\-2\\1\end{matrix}\right)$$

The point $P$ has position vector:

$$\left(\begin{matrix}2\\1\\3\end{matrix}\right)$$

1. Show that $P$ does not line on $l$
2. Given that a circle, centre $P$, intersects $l$ at points $A$ and $B$, and that $A$ has position vector:

$$A=\left(\begin{matrix}0\\-3\\6\end{matrix}\right)$$

Find the position vector of $B$.

**9B Part 1 3D Planes Introduction**

1. Find, in the form $r=a+λb+μc$, an equation of the plane that passes through the points $A(2,2,-1)$, $B(3,2,-1)$ and $C(4,3,5)$



1. Verify that the point $P$ with position vector $\left(\begin{matrix}2\\2\\-1\end{matrix}\right)$ lies in the plane with vector equation:

$$r=\left(\begin{matrix}3\\4\\-2\end{matrix}\right)+λ\left(\begin{matrix}2\\1\\1\end{matrix}\right)+μ\left(\begin{matrix}1\\-1\\2\end{matrix}\right)$$

**9B Part 2 Cartesian 3D Planes**

2D notes:

1. The straight line graph has normal vector $\left(\begin{matrix}-1\\4\end{matrix}\right)$ and passes through $(2,3)$. Find the equation of the line.

3D notes:

1. The plane $Π$ is perpendicular to the normal vector $n=3i-2j+k$ and passes through the point P with position vector $8i+4j-7k$. Find a Cartesian equation of $Π$.



**9C Scalar Products & Angles Between Lines**

1. Given that $a=\left(\begin{matrix}8\\-5\\-4\end{matrix}\right)$ and $b=\left(\begin{matrix}5\\4\\-1\end{matrix}\right)$.
2. Find $a.b$
3. Find the angle between $a$ and $b$, giving your answer in degrees to 1 decimal place
4. Given that the vectors $a=2i-6j+k$ and $b=5i+2j+λk$are perpendicular, find the value of $λ$**.**
5. Given that $a=-2i+5j-4k$ and $b=4i-8j+5k$, find a vector which is perpendicular to both $a$ and $b$.
6. The points $A$, $B$ and $C$ have coordinates $(2,-1,1)$, $(5,1,7) $and $(6,-3,1) $respectively.
7. Find $\vec{AB}.\vec{AC}$
8. Hence, or otherwise, find the area of triangle $ABC$

**9D Acute Angles Between Lines & Planes**

1. The lines $l\_{1}$ and $l\_{2}$ have vector equations:

$$r=\left(2i+j+k\right)+t(3i-8j-k)$$

and

$$r=\left(7i+4j+k\right)+s(2i+2j+3k)$$

Given that $l\_{1}$ and $l\_{2}$ intersect, find the size of the acute angle between the lines, to 1 decimal place.

r.n = k for equation of a plane notes

1. The plane $Π$ passes through the point $A$ and is perpendicular to the vector $n$.

Given that $\vec{OA}=\left(\begin{matrix}2\\3\\-5\end{matrix}\right)$ and $n=\left(\begin{matrix}3\\1\\-1\end{matrix}\right)$, with O being the origin, find an equation of the plane:

1. In scalar product form
2. In Cartesian form
3. Find the acute angle between the line $l$ with equation:

$$r=2i+j-5k+λ(3i+4j-12k)$$

and the plane with equation:

$$r.\left(2i-2j-k\right)=2$$

1. Find the acute angle between the planes with equations $r.\left(\begin{matrix}4\\4\\-7\end{matrix}\right)=13$ and $r.\left(\begin{matrix}7\\-4\\4\end{matrix}\right)=6$.

**9E Points of Intersection**

1. The lines $l\_{1}$ and $l\_{2}$ have vector equations:

$$r=\left(\begin{matrix}3\\1\\1\end{matrix}\right)+λ\left(\begin{matrix}1\\-2\\-1\end{matrix}\right)$$

and

$$r=\left(\begin{matrix}0\\-2\\3\end{matrix}\right)+μ\left(\begin{matrix}-5\\1\\4\end{matrix}\right)$$

Show that the lines intersect, and find their point of intersection.

1. Find the coordinates of the point of intersection of the line $l$ and the plane $Π$ where $l$ has equation:

$$r=-i+j-5k+λ(i+j+2k)$$

And $Π$ has equation:

$$r.\left(i+2j+3k\right)=4$$

1. The lines $l\_{1}$ and $l\_{2}$ have equations:

$$\frac{x-2}{4}=\frac{y+3}{2}=z-1$$

and

$$\frac{x+1}{5}=\frac{y}{4}=\frac{z-4}{-2}$$

respectively.

Prove that $l\_{1}$ and $l\_{2}$ are skew.

**9F Part 1 Perpendicular Distances with Lines**







1. Show that the shortest distance between the parallel lines with equations:

$$r=i+2j-k+λ(5i+4j+3k)$$

and

$$r=2i+k+μ(5i+4j+3k)$$

is $\frac{21\sqrt{2}}{10}$

1. The lines $l\_{1}$ and $l\_{2}$ have equations:

$$r=\left(\begin{matrix}1\\0\\0\end{matrix}\right)+λ\left(\begin{matrix}0\\1\\1\end{matrix}\right)$$

$$r=\left(\begin{matrix}-1\\3\\-1\end{matrix}\right)+μ\left(\begin{matrix}2\\-1\\-1\end{matrix}\right)$$

Find the shortest distance between these two lines.

1. The line $l$ has equation:

$$\frac{x-1}{2}=\frac{y-1}{-2}=\frac{z+3}{-1}$$

The point $A$ has coordinates $(1,2,-1)$

1. Find the shortest distance between $A$ and $l$.
2. Find a Cartesian equation of the line that is perpendicular to $l$, and passes through $A$.

**9F Part 2 Perpendicular Distances with Lines & Planes**



1. Find the perpendicular distance from the point with coordinates $(3,2,-1)$ to the plane with equation $2x-3y+z=5$
2. The plane $Π$ has equation:

$$r.\left(i+2j+2k\right)=5$$

The point $P$ has coordinates:

$$(1,3,-2)$$

1. Find the shortest distance between $P$ and $Π$
2. The point $Q$ is a reflection of $P$ in $Π$. Find the coordinates of $Q$.
3. The line $l\_{1}$ has equation:

$$\frac{x-2}{2}=\frac{y-4}{-2}=\frac{z+6}{1}$$

The plane $Π$ has equation:

$$2x-3y+z=8$$

The line $l\_{2}$ is a reflection of $l\_{1}$ in the plane $Π$. Find a vector equation of the line $l\_{2}$.