Modelling

[June 2013 (Withdrawn) Q8]

(a) Express $9\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the exact value of R and give the value of α to 4 decimal places. **(3)**

(b) (i) State the maximum value of $9\cos\theta - 2\sin\theta$

(ii) Find the value of θ, for $0 < \theta < 2\pi$, at which this maximum occurs. **(3)**

Ruth models the height H above the ground of a passenger on a Ferris wheel by the equation

$$H = 10 - 9\cos\left(\frac{\pi t}{5}\right) + 2\sin\left(\frac{\pi t}{5}\right)$$

where H is measured in metres and t is the time in minutes after the wheel starts turning.

(c) Calculate the maximum value of H predicted by this model, and the value of t, when this maximum first occurs. Give your answers to 2 decimal places. **(4)**

(d) Determine the time for the Ferris wheel to complete two revolutions. **(2)**

When trigonometric equations are in the form $Rsin(ax \pm b)$ or $Rcos(ax \pm b)$, they can be used to model various things which have an oscillating behaviour, e.g. tides, the swing of a pendulum and sound waves.

Test Your Understanding

[June 2010 Q7] 2. (*a*) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places. **(3)**

(*b*) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

(ii) Find the value of θ, for $0 \leq \theta < \pi$, at which this maximum occurs. **(3)**

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2 \sin\left(\frac{4\pi t}{25}\right) - 1.5 \cos\left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

(*c*) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs. **(3)**

(*d*) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres. **(6)**

Tip: Reflect carefully on the substitution you use to allow (bii) to match your identity in (a). θ =?

Textbook Exercise 7G Page 190