$a \sin\theta + b \cos\theta$

Q Put $3 \sin x + 4 \cos x$ in the form $R \sin(x + \alpha)$ giving α in degrees to 1dp.

STEP 1: Expanding:

STEP 2: Comparing coefficients:

STEP 3: Using the fact that $R^2 \sin^2 \alpha + R^2 \cos^2 \alpha = R^2$:

STEP 4: Using the fact that $\frac{R \sin \alpha}{R \cos \alpha} = \tan \alpha$:

STEP 5: Put values back into original expression.

Test Your Understanding

Q Put $\sin x + \cos x$ in the form $R\sin(x + \alpha)$ giving α in terms of π.

Q Put $\sin x - \sqrt{3}\cos x$ in the form $R\sin(x - \alpha)$ giving α in terms of π.

Q Put $2\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $0 < \alpha < 90°$
Hence solve, for $0 < \theta < 360$, the equation $2\cos\theta + 5\sin\theta = 3$

Tip: This is an exam favourite!

Q (Without using calculus), find the maximum value of $12\cos\theta + 5\sin\theta$, and give the smallest positive value of θ at which it arises.

Expression	Maximum	(Smallest) θ at max
$20\sin\theta$		
$5-10\sin\theta$		
$3\cos(\theta+20°)$		
$\dfrac{2}{10+3\sin(\theta-30)}$		

Further Test Your Understanding

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4. (*a*) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(*b*) $$p(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \quad 0 \leq \theta \leq 2\pi.$$

Calculate

(i) the maximum value of $p(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)

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