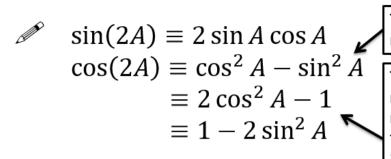
Double Angle Formulae

Double-angle formula allow you to halve the angle within a trig function.



This first form is relatively rare.

Tip: The way I remember what way round these go is that the cos on the RHS is 'attracted' to the cos on the LHS, whereas the sin is pushed away.

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

These are all easily derivable by just setting A=B in the compound angle formulae. e.g.

$$sin(2A) = sin(A + A)$$

$$= sin A cos A + cos A sin A$$

$$= 2 sin A cos A$$

Examples

[Textbook] Use the double-angle formulae to write each of the following as a single trigonometric ratio.

a)
$$\cos^2 50^\circ - \sin^2 50^\circ$$

b)
$$\frac{2\tan\left(\frac{\pi}{6}\right)}{1-\tan^2\left(\frac{\pi}{6}\right)}$$

c)
$$\frac{4 \sin 70^{\circ}}{\sec 70^{\circ}}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Examples

[Textbook] Given that $x=3\sin\theta$ and $y=3-4\cos2\theta$, eliminate θ and express y in terms of x.

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Given that $\cos x = \frac{3}{4}$ and x is acute, find the exact value of (a) $\sin 2x$ (b) $\tan 2x$

Note: This question is an example of turning a set of **parametric equations** into a single **Cartesian** one. You will cover this in the next chapter.