Double Angle Formulae

Double-angle formula allow you to halve the angle within a trig function.

$$\sin(2A) \equiv 2\sin A\cos A$$
$$\cos(2A) \equiv \cos^2 A - \sin^2 A$$
$$\equiv 2\cos^2 A - 1$$
$$\equiv 1 - 2\sin^2 A$$

This first form is relatively rare.

Tip: The way I remember what way round these go is that the cos on the RHS is 'attracted' to the cos on the LHS, whereas the sin is pushed away.

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

These are all easily derivable by just setting $A = B$ in the compound angle formulae. e.g.

$$\sin(2A) = \sin(A + A)$$
$$= \sin A\cos A + \cos A\sin A$$
$$= 2\sin A\cos A$$

Examples

[Textbook] Use the double-angle formulae to write each of the following as a single trigonometric ratio.

a) $\cos^2 50° - \sin^2 50°$

b) $\frac{2\tan\left(\frac{\pi}{6}\right)}{1-\tan^2\left(\frac{\pi}{6}\right)}$

c) $\frac{4\sin 70°}{\sec 70°}$

$$\sin 2x = 2\sin x\cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Examples

[Textbook] Given that $x = 3\sin\theta$ and $y = 3 - 4cos2\theta$, eliminate θ and express y in terms of x.

$$\sin 2x = 2\sin x\cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Note: This question is an example of turning a set of **parametric equations** into a single **Cartesian** one. You will cover this in the next chapter.

Given that $\cos x = \frac{3}{4}$ and x is acute, find the exact value of
(a) $\sin 2x$ (b) $\tan 2x$

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